壓變黏度效應力對於指數型曲面圓盤負荷能力與響應時間之影響

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摘　要

在 Barus [9] 實驗公式之基礎下，本文考慮壓變黏度之效應，進而延伸 Murti [7]對於指數型曲面圓盤負荷能力與響應時間之研究。若與 Murti[7] 所得定黏度潤滑劑之結果比較，壓變黏度之效應可以增進指數型曲面圓盤之擠壓性能。藉由潤滑劑之壓變參數與曲面圓盤之曲率參數的變化，本研究所得負荷能力與響應時間之性能曲線可以提工程業界之參考與應用。

關鍵詞: 指數型曲面圓盤，壓變黏度效應，負荷能力，響應時間

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PIEZO-VISCOUS EFFECTS ON THE LOAD CAPACITY AND RESPONSE TIME OF EXPONENTIAL CURVED CIRCULAR DISKS

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Abstract

Based upon the experimental formula of Barus [9], the study of squeeze film problems between exponential curved circular disks of Murti [7] is extended by considering the piezo-viscous effects of lubricants. Comparing with the case of iso-viscous lubricants of Murti [7], the effects of exponential dependency of viscosity improve the performance characteristics of exponential curved circular disks. The results through the variation of the piezo-viscous parameter and the exponential curvature parameter provide useful references and applications for industrial engineering.

Key Words: exponential curved disks, piezo-viscous effects, load capacity, response time

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1. INTRODUCTION

The squeeze film pressure is built up when two lubricated plates approach each other with a squeezing velocity. A finite response time is then required to squeeze the lubricant out of the plates. As a results, the lubricant film within the plates acts as a useful cushion. Squeeze film plates with different configurations receive great attention in many areas of applications such as human joints, mechanical components, matching gears, clutch plates, etc. By assuming an iso-viscous property (i.e., the lubricant viscosity $\eta = \text{constant}$), many investigators have studied the circular squeeze film problems, such as the squeeze film characteristics between parallel circular disks by Pinkus and Sternlicht [1], Murti [2], Fuller [3] and Hamrock [4]; the squeeze film performances between parallel annular disks by Allen and McKillop [5] and Wu [6]; and the squeeze film behavior between curved disks by Murti [7] and Bhat and Hingu [8]. In accordance with the experimental study by Barus [9], the variation of the lubricant viscosity η grows exponentially with the pressure p is suggested through the relationship.

$$\eta = \eta_0 \exp(\xi \cdot p) \tag{1}$$

where η_0 denotes the isothermal viscosity at the ambient pressure, and ξ represents the piezo-viscous coefficient. Based upon this consideration, Chen [10] investigates the elastohydrodynamic problems in the inlet region of line contacts. It is found that the piezo-viscous effects on the film thickness are not negligible. Elsharkawy [11] analyzes the lubrication performances of externally pressurized circular porous bearings. It is shown that the piezo-viscous effects have significant influences on the pressure profiles and the load-carrying capacity. However, the study for squeeze film problems between exponential curved circular disks by considering the piezo-viscous effects is still absent. Since the exponential curved circular mechanism is important in engineering practice as Murti [7] and Bhat and Hingu [8], further studies are motivated.

In the present paper, we extend the study of squeeze film problems between exponential curved circular disks of Murti [7] by considering the piezo-viscous effects of lubricants. A closed-form analytical solution of the film pressure will be derived by solving the nonlinear modified Reynolds equation. Comparing with the case of iso-viscous lubricants, the piezo-viscous effects on the squeeze film characteristics of

curved circular disks are presented and discussed in terms of the load capacity and the response time through the variation of the piezo-viscous parameter and the exponential curvature parameter.

2. FORMULATION AND ANALYSIS

Figure 1 shows the squeeze film configuration between exponential curved circular disks lubricated with an incompressible isothermal Newtonian lubricant considering the piezo-viscous effects, in which the upper curved disk is moving towards the lower disk with a squeezing velocity: $-dh_m/dt$. The radius of the disk is a and the minimum film thickness is h_m. The film thickness h is symmetric about the $z-$axis and is described by an exponential function as Murti [7].

$$h = h_m \cdot \exp(-\beta \cdot r^2 / a^2) \tag{2}$$

In this expression, β is the exponential curvature parameter defining the film shapes. For $\beta < 0$, convex films are generated; for $\beta = 0$, plane-disk films are recovered; and for $\beta > 0$, concave films are developed between the disks. According to the derivation of Lu and Lin [12], the modified Reynolds equation considering the piezo-viscous effects for the exponential curved circular disks is

$$\frac{1}{r}\frac{d}{dr}\left(\frac{rh^3}{\eta}\cdot\frac{dp}{dr}\right) = 12\frac{dh_m}{dt} \tag{3}$$

It is noted that when the piezo-viscous coefficient $\xi = 0$, then $\eta =$ constant, and the above modified Reynolds equation reduces to the case of iso-viscous lubricants by Murti [7]. In order to analyze the problems conveniently, the non-dimensional variables and parameters are introduced as follows.

$$R = \frac{r}{a},\quad H_m = \frac{h_m}{h_{m0}},\quad H = \frac{h}{h_{m0}},\quad P = \frac{p h_{m0}^{\ 3}}{\eta_0 a^2(-dh_m/dt)} \tag{4}$$

$$\gamma = \frac{\xi \cdot \eta_0 a^2(-dh_m/dt)}{h_{m0}^{\ 3}} \tag{5}$$

where γ represents the piezo-viscous parameter characterizing the effects of pressure varying with the viscosity. As a result, the film thickness, the modified Reynolds equation considering the piezo-viscous effects, and the pressure boundary conditions can be written in non-dimensional form.

$$H = H_m \cdot \exp(-\beta \cdot R^2) \tag{6}$$

$$\frac{1}{R}\cdot\frac{d}{dR}\left\{R\cdot\frac{H^3}{\exp(\gamma\cdot P)}\cdot\frac{dP}{dR}\right\}=-12 \tag{7}$$

$$\frac{dP}{dR}=0 \quad \text{at} \quad R=0 \tag{8}$$

$$P=0 \quad \text{at} \quad R=1 \tag{9}$$

The non-dimensional Reynolds equation is a highly nonlinear differential equation. However, integrating the differential equation twice and applying the pressure boundary conditions, one can derive the solution for the squeeze film pressure.

$$P=-\frac{1}{\gamma}\cdot\ln\left[1+\frac{\gamma}{\beta H_m}\cdot(e^{3\beta\cdot R^2}-e^{3\beta})\right] \tag{10}$$

Integrating the squeeze film pressure over the film region, one can obtain the load-carrying capacity.

$$W=\int_{r=0}^{+a} p\cdot 2\pi r dr \tag{11}$$

Introducing the non-dimensional form yields:

$$L=\frac{W{h_{m0}}^3}{2\pi\mu_0 a^4(-dH_m/dt)}=\int_{r^*=0}^{+1} P\cdot RdR \tag{12}$$

After performing the integration, the load-carrying capacity is obtained.

$$L=-\frac{1}{\gamma}\cdot\int_{R=0}^{+1}\ln\left[1+\frac{\gamma}{\beta H_m}\cdot(e^{3\beta\cdot R^2}-e^{3\beta})\right]\cdot RdR \tag{13}$$

Introducing the non-dimensional response time:

$$T=\frac{W{h_{m0}}^2}{2\pi\mu_0 a^4}\cdot t \tag{14}$$

one can obtain the differential equation governing the variation of non-dimensional response time with the non-dimensional film thickness.

$$\frac{dH_m}{dt^*}=\gamma\cdot\left\{\int_{R=0}^{+1}\ln\left[1+\frac{\gamma}{\beta {H_m}^3}\cdot(e^{3\beta\cdot R^2}-e^{3\beta})\right]\cdot RdR\right\}^{-1} \tag{15}$$

Integrating the differential equation and applying the initial condition: $H_m(T=0)=1$, one can derive the relationship for the response time varying with film thickness.

$$T=-\frac{1}{\gamma}\cdot\int_{H_m}^{H_m=1}\int_{R=0}^{+1}\ln\left[1+\frac{\gamma}{\beta H_m}\cdot(e^{3\beta\cdot R^2}-e^{3\beta})\right]\cdot RdR\cdot dH_m \tag{16}$$

The values of the non-dimensional load capacity and the non-dimensional response time can be numerically obtained by using the Gaussian-Quadrature method by Faires and Burden [13].

3. RESULTS AND DISCUSSION

Based on the above formulation and analysis, a closed-form analytical solution has been derived for the squeeze film problems between exponential curved circular disks with piezo-viscous effects. Three special cases can be recovered from some specific values of the exponential curvature parameter β defined in equation (2) and the piezo-viscous parameter γ defined in equation (5). Case 1: $\beta = 0$, $\gamma = 0$. It is the plane-disk squeeze film problem using the iso-viscous lubricants by Fuller [3] and Hamrock [4]. Taking the limit of equation (13) gives their results.

$$\lim_{\gamma \to 0, \beta \to 0} L = \frac{0.75}{H_m^{\ 3}} \tag{17}$$

Case 2: $\beta \neq 0$, $\gamma = 0$. It is the curved-disk squeeze film problem using the iso-viscous lubricants. Taking the limit of equation (13) yields the results derived by Murti [**7**].

$$\lim_{\beta \neq 0, \gamma \to 0} L = -\frac{1}{6\beta^2 H_m^{\ 3}} \cdot [e^{3\beta} \cdot (1 - 3\beta) - 1] \tag{18}$$

Case 3: $\beta = 0$, $\gamma \neq 0$. It is the plane-disk squeeze film problem considering the piezo-viscous effects. Taking the limit of equation (13) one can obtain the load-carrying capacity.

$$\lim_{\beta \to 0, \gamma \neq 0} L = -\frac{1}{\gamma} \cdot \int_{R=0}^{+1} \ln\left[1 + \frac{3\gamma}{H_m^{\ 3}} \cdot (R^2 - 1)\right] \cdot RdR \tag{19}$$

Figure 2 presents the variation of the non-dimensional load-carrying capacity L with the non-dimensional minimum film height H_m for different values of β and γ. Comparing with the case of iso-viscous lubricants, the squeeze film disks considering the piezo-viscous effects $(\gamma = 0.005)$ are observed to result in higher values of the load-carrying capacity. Changing the convex film shape $(\beta = -0.2)$ to the plane film shape $(\beta = 0)$ and to the concave film shape $(\beta = +0.2)$ provides further increments of the load resulting from the influences of piezo-viscosity.

Figure 3 describes the effect of variation of γ on the non-dimensional response time T for different β under the non-dimensional minimum film heights $H_m = 0.4$ and $H_m = 0.3$. The response time is the time required for the height between the disk surfaces to be reduced to a given film height. For the case of final minimum film height $H_m = 0.4$, the response time T is observed to increase slightly with the value of γ. However, for the case of final minimum film height is $H_m = 0.3$, longer values of the

response time are obtained for larger values of the piezo-viscous parameter and the exponential curvature parameter. On the whole, the disks considering the piezo-viscous effects are observed to provide an improvement in the squeeze film characteristics and result in longer exponential curved circular squeeze film life.

4. CONCLUSIONS

A closed-form solution for the film pressure has been derived for the squeeze film problems between exponential curved circular disks considering the variation of the viscosity growing exponentially with film pressure. Comparing with the case of iso-viscous lubricants, the piezo-viscous effects provide an increased load capacity and a longer response time, and therefore result in longer operating life for exponential curved circular squeeze-film disks.

REFERENCES

1. Pinkus, O. and Sternlicht, B., Theory of Hydrodynamic Lubrication, McGraw-Hill, New York (1961).
2. Murti, P.R.K., “Squeeze-film Behavior in Porous Circular Disks,” ASME Journal of Lubrication Technology, Vol. 96, pp. 206-209 (1974).
3. Fuller, D.D., Theory and Practice of Lubrication for Engineers, John Wiley & Sons, Inc., New York (1984).
4. Hamrock, B.J., Fundamentals of Fluid Film Lubrication, McGraw-Hill, New York (1994).
5. Allen, C.W. and McKillop, A.A., “An Investigation of the Squeeze Film between Rotating Annuli,” ASME Journal of Lubrication Technology, Vol. 92, pp. 435-441 (1970).
6. Wu, H., “Squeeze Film Behavior for Porous Annular Disks,” ASME Journal of Lubrication Technology, Vol. 92, pp. 593-596 (1970).
7. Murti, P.R.K., “Squeeze Films in Curved Circular Plates,” ASME Journal of Lubrication Technology, Vol. 97, pp. 650-652 (1975).
8. Bhat, M.V. and Hingu, J.V., “Squeeze Films in Curved Porous Circular Plates,” Wear, Vol. 52, pp. 193-196 (1979).
9. Barus, C., “Isothermals, Isopiestics, and Isometrics Relative to Viscosity,”

American Journal of Science, Vol. 45, pp. 87-96 (1893).

10. Cheng, H.S., “Isothermal Elastohydrodynamic Theory for the Full Range of Pressure-viscosity Coefficient,” ASME Journal of Lubrication Technology, Vol. 94, pp. 35-43 (1972).
11. Elsharkawy, A.A., Kassab, S.Z. and Nassar, M.M., “Lubrication Analysis of Externally Pressurized Circular Porous Bearings,” Applied Mathematical Modelling, Vol. 20, pp. 870-876 (1996).
12. Lu, R.F. and Lin, J.R., “A Theoretical Study of Combined Effects of Non-Newtonian Rheology and Viscosity-pressure Dependence in the Sphere-plate Squeeze-film System,” Tribology International, Vo. 40, pp. 125-131 (2007).
13. Faires, J.D. and Burden, R., Numerical Methods, Brooks/CPC, California (1998).

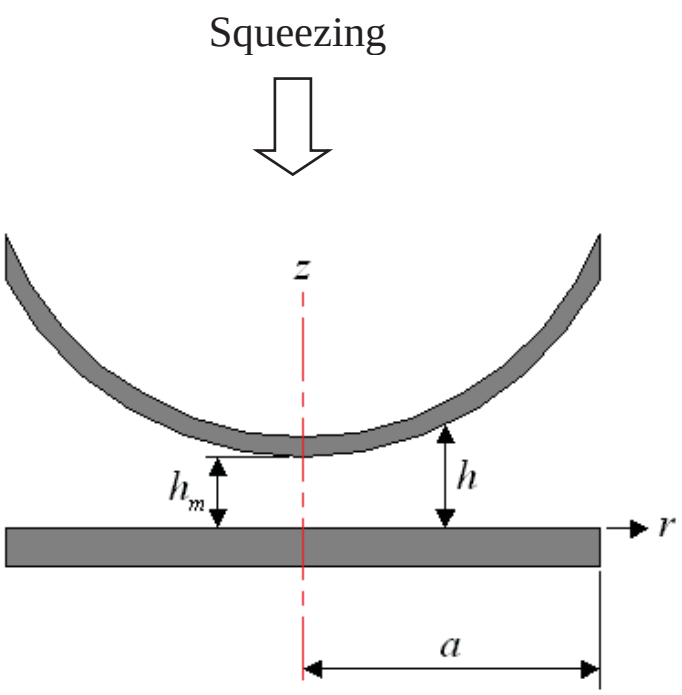


Figure 1 Squeeze film configuration between exponential curved circular disks considering the piezo-viscous effects

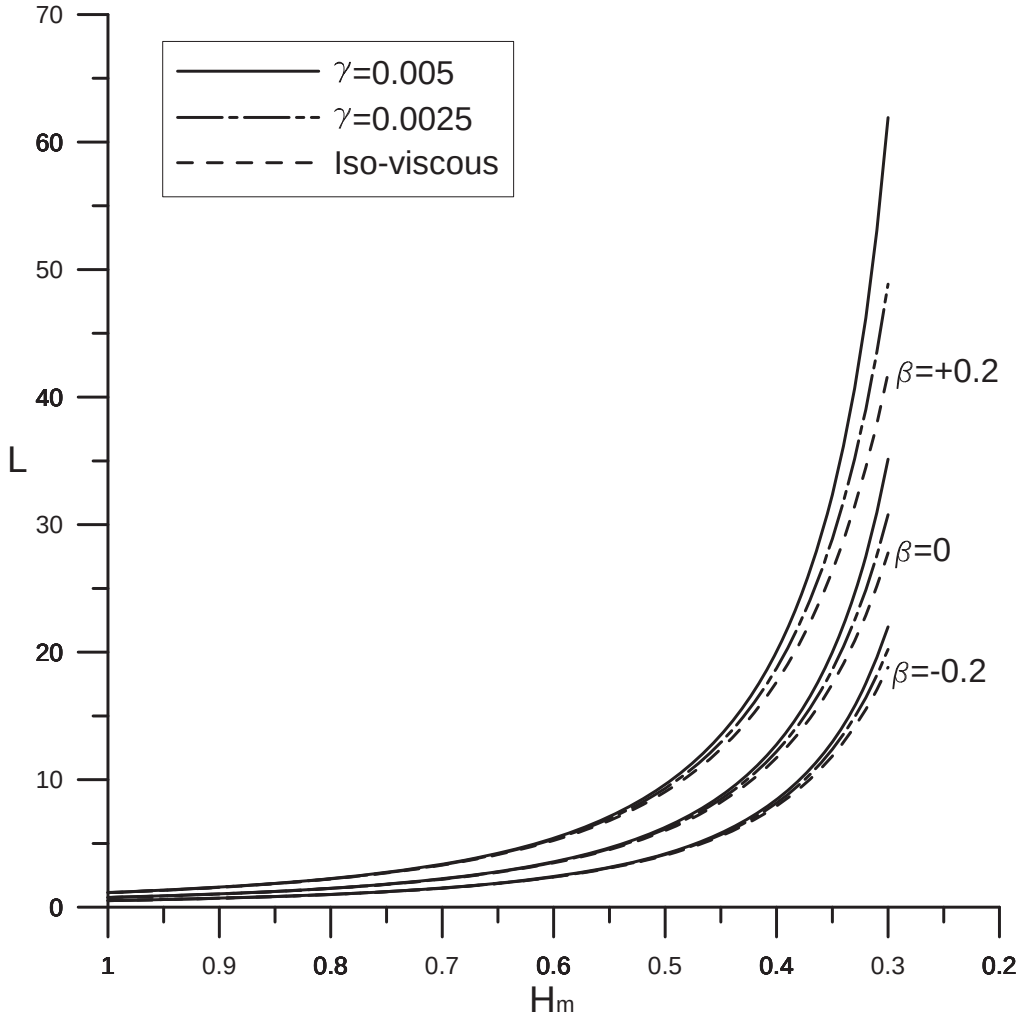
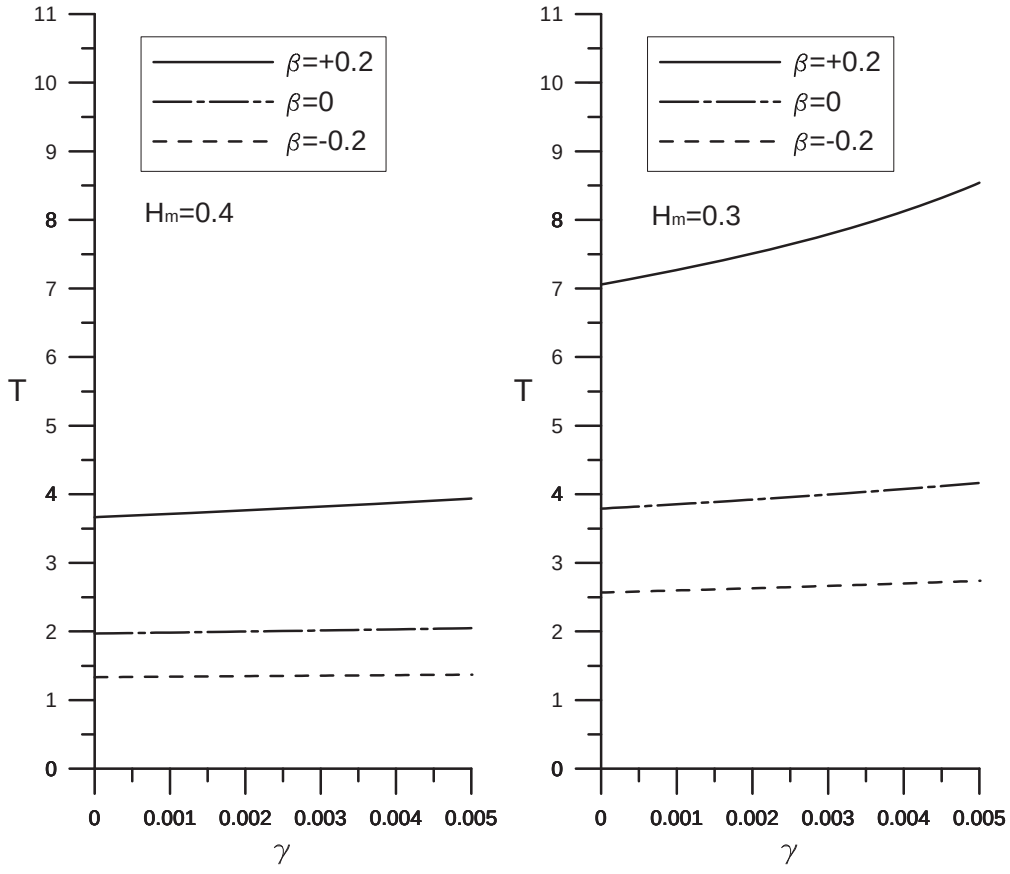


Figure 2 Variation of the load capacity L with the minimum film height H_m for different β and γ



(a) $H_m = 0.4$ (b) $H_m = 0.3$

Figure 3 Effect of variation of γ on the response time T for different β under the minimum film heights $H_m = 0.4$ and $H_m = 0.3$