黏度壓力相依性對於餘弦型態凸形曲面板間擠壓性能之影響

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摘　要

本文主要在探討潤滑劑黏度隨壓力變化之相依性效應，對於餘弦型態凸形曲面板間薄膜擠壓性能之影響。根據所得結果顯示，若與定黏度潤滑劑之情況比較，黏度壓力相依性之效應提供了餘弦凸形曲面擠壓板間較高之擠壓膜壓力與負荷能力，同時亦提供了較長之擠壓膜響應時間。此種研究成果，可提供工程師於設計薄膜曲面板擠壓機制時一個有用之參考訊息。

關鍵詞: 凸形曲面板，餘弦函數，黏度壓力相依性，擠壓膜

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EFFECTS OF PRESSURE-DEPENDENT VISCOSITY ON THE SQUEEZE FILM CHARACTERISTICS OF COSINE-FORM CONVEX CURVED PLATES

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Abstract

The effects of pressure-dependent viscosity on the squeeze film characteristics between convex curved plates of a cosine form are presented in the present study. Comparing with case of iso-viscous lubricants, the effects pressure-dependent viscosity signify an increase in the values of the squeeze film pressure, the load capacity and the elapsed time for the cosine-form convex curved plates. It provides useful information in designing the mechanisms of squeeze film plates for engineering applications.

Key Words: convex curved plates, cosine function, pressure-dependent viscosity, squeeze film

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1. INTRODUCTION

Squeeze film mechanisms between curved surfaces are widely applied in industrial practice such as the approaching gears, cam and followers, hydraulic damping devices, and artificial joints. In the literature, the squeeze film behaviors between curved plates have been analyzed by assuming the lubricant with an IV (iso-viscous) property, such as the rectangular sine-curved films by Hay [1], the circular exponential-curved films by Murti [2], the sphere-socket films by Pinkus and Sternlicht [3] and Lin [4], the cylinder-plate films by Hamrock [5] and Lin *et al.* [6] and the sphere-plate films by Lin [7]. On the other hand, an experimental study carried out by Barus [8] shows that under the isothermal condition the viscosity of the lubricant varies with the film pressure. He suggested that the lubricant viscosity η grows with the pressure described by an exponential equation.

$$\eta = \eta_0 e^{ap} \tag{1}$$

In this equation, p is the film pressure, a is the pressure-dependent constant and η_0 is the lubricant viscosity at atmospheric pressure. Since the squeeze film pressure is generally increasing with decreasing film thickness, the PDV (pressure-dependent viscosity) should be considered especially for upper plate achieves a lower squeeze film height. Therefore, a further study between curved squeeze film surfaces including the effects of PDV in equation (1) is of interest.

In the present study, a squeeze film mechanism of convex curved plates of a cosine form is proposed. Comparing with case of iso-viscous lubricants, the effects of PDV on the squeeze film characteristics are presented and discussed through the variation of the PDV parameter, the amplitude ratio and the non-dimensional minimum film thickness of the cosine-form convex plates.

2. ANALYSIS

Figure 1 shows the squeeze film geometry between convex curved plates with a cosine form considering the effects of PDV. The film thickness h for the squeeze film can be generated by the form of a cosine function.

$$h = h_m + d \cdot [1 - \cos(\frac{\pi}{L} x)] \tag{2}$$

In the equation, L is the length of the plates, d is the amplitude of the cosine

function, and h_m is the minimum film thickness. At the central position: $x=0$, the film thickness is equal to the minimum film thickness, h_m. At the edge position: $x=L/2$, the film thickness is equal to the sum of the minimum film thickness and the amplitude, h_m+d. The advantage of the proposed equation (1) is that: for a fixed length L, one can generate different sizes of convex curved surfaces by varying the value of amplitudes for the cosine function. According to the study of combined effects of non-Newtonian rheology and pressure-dependent viscosity in the sphere-plate system by Lu and Lin [9], the modified Reynolds equation considering the effects of PDV can be written as:

$$\frac{d}{dx}\left(\frac{h^3}{\eta}\cdot\frac{dp}{dx}\right)=12\dot{h}_m \tag{3}$$

In this equation, $\dot{h}_m$ denotes dh_m/dt, where t is the time. In order to analyze the problem conveniently, the non-dimensional variables and parameters are introduced.

$$x^*=\frac{x}{L},\quad h_m{}^*=\frac{h_m}{h_{m0}},\quad h^*=\frac{h}{h_{m0}},\quad p^*=\frac{ph_{m0}{}^3}{\mu_0 L^2(-\dot{h}_m)} \tag{4}$$

Thereafter, the non-dimensional modified Reynolds equation and the non-dimensional film thickness can be expressed as:

$$\frac{d}{dx^*}\left\{h^{*3}e^{-Pp^*}\frac{dp^*}{dx^*}\right\}=-12 \tag{5}$$

$$h^*=h_m{}^*+A\cdot[1-\cos(\pi x^*)] \tag{6}$$

In these equations, A is the amplitude ratio of the cosine-form convex curved plates and P is the PDV parameter, respectively.

$$A=\frac{d}{h_{m0}} \tag{7}$$

$$P=\frac{\mu_0 aL^2(-\dot{h}_m)}{h_{m0}{}^3} \tag{8}$$

The pressure conditions are: $p^*=0$ at $x^*=\pm 1/2$ and $dp^*/dx^*=0$ at $x^*=0$. Integrating (5) and using the pressure conditions, one can obtain

$$p^*=-\frac{1}{P}\cdot\ln\left\{1-\int_{x^*}^{1/2}\frac{12P\cdot x^*}{\{h_m{}^*+A\cdot[1-\cos(\pi x^*)]\}^3}dx^*\right\} \tag{9}$$

Integrating the film pressure, one can obtain the load-carrying capacity.

$$W=\int_{x=-L/2}^{+L/2}p\cdot Ddx \tag{10}$$

where D denotes the width of the curved plates. Introducing a non-dimensional form gives

$$W^* = \frac{W h_{m0}^{\ 3}}{\eta_0 L^3 D(-\dot{h}_m)} = \int_{x^*=-1/2}^{1/2} p^* \cdot dx^* \tag{11}$$

After performing the integration, one can obtain the non-dimensional load capacity.

$$W^* = -\frac{1}{P} \cdot \int_{x^*=-1/2}^{1/2} \ln\left[1 - \int_{x^*}^{x^*=1/2} \frac{12P \cdot x^*}{\{h_m^{\ *} + A \cdot [1-\cos(\pi x^*)]\}^3} dx^*\right] \cdot dx^* \tag{12}$$

Introduce a non-dimensional time as:

$$t^* = \frac{W h_{m0}^{\ 2}}{\eta_0 L^3 D} \cdot t \tag{13}$$

Then:

$$\dot{h}_m = \frac{dh_m}{dt} = \frac{W h_{m0}^{\ 3}}{\eta_0 L^3 D} \cdot \frac{dh_m^{\ *}}{dt^*} \tag{14}$$

Substituting into (11) results in

$$\frac{dh_m^{\ *}}{dt^*} = \frac{P}{\int_{x^*=-1/2}^{1/2} \ln\left[1 - \int_{x^*}^{x^*=1/2} \frac{12P \cdot x^*}{\{h_m^{\ *} + A \cdot [1-\cos(\pi x^*)]\}^3} dx^*\right] dx^*} \tag{15}$$

The initial condition for the minimum film thickness is: $h_m^{\ *}(t^* = 0) = 1$. After integrating the differential equation, the elapsed time required for the upper curved plate to approach the lower plate can be derived.

$$t^* = -\frac{1}{P} \int_{h_m^{\ *}}^{1} \int_{x^*=-1/2}^{1/2} \ln\left[1 - \int_{x^*}^{1/2} \frac{12P \cdot x^*}{\{h_m^{\ *} + A \cdot [1-\cos(\pi x^*)]\}^3} dx^*\right] dx^* dh_m^{\ *} \tag{16}$$

Using the numerical method of integration, the film pressure (9), the load capacity (12) and the elapsed time (16) can be calculated.

3. RESULTS AND DISCUSSION

Figure 2 shows the film pressure p^* as a function of the horizontal coordinate x^* for different values of the amplitude ratio A and the PDV parameter P. From the definition of the geometry in equation (6), the cosine-form convex curved plates reduce to the parallel flat plates as the value of the amplitude ratio $A = 0$. Observing the displayed results, the peak values of the film pressure for the cosine-form convex curved plates ($A = 0.1$, $A = 0.2$) are lower than those of the parallel flat plates. Under the amplitude ratio $A = 0.2$, the PDV effects ($P = 0.002$) results in a higher film

pressure as compared to the IV (iso-viscous) case. Increasing the value of the PDV parameter ($P = 0.004$) yields further increments of the film pressure. When the amplitude ratio decreases down to $A = 0.1$ or the situation of flat plates ($A = 0$), the increased amounts of the film pressure due to the effects of PDV are more emphasized. Figure 3 shows the load capacity W^* as a function of the amplitude ratio A for different values of the minimum film thickness h_m^* and the PDV parameter P. At the film thickness $h_m^* = 0.3$, the value of W^* is seen to decrease with increasing A. Comparing with the IV case, visible increments of the load capacity resulting from the effects of PDV ($P = 0.002$, $P = 0.004$) are observed. It is also shown that decreasing the film thickness increases the influences of PDV on the load capacity of the convex curved plates. Figure 4 presents the minimum film height h_m^* as a function of the elapsed time t^* for different values of the PDV parameter P under the amplitude ratio $A = 0.1$. Comparing with the IV case, the PDV effects ($P = 0.002$) are observed to yield a longer elapsed time for the convex plates. Increasing the PDV parameter ($P = 0.004$) increases the longer time for the upper curved plate required to achieve a prescribed film height. On the whole, the elapsed times for the cosine-form convex curved plates are lengthened by considering the effects of PDV of lubricants.

4. CONCLUSIONS

The influences of pressure-dependent viscosity are significantly on the squeeze films between convex plates of a cosine form. Comparing with the iso-viscous case, the effects of pressure-dependent viscosity provide an increase in the load capacity and the elapsed time for the cosine-form plates. The quantitative influences of pressure-dependent viscosity are further highlighted for the cosine-form convex plates with a larger pressure-dependent viscosity parameter, and a smaller amplitude ratio and film thickness.

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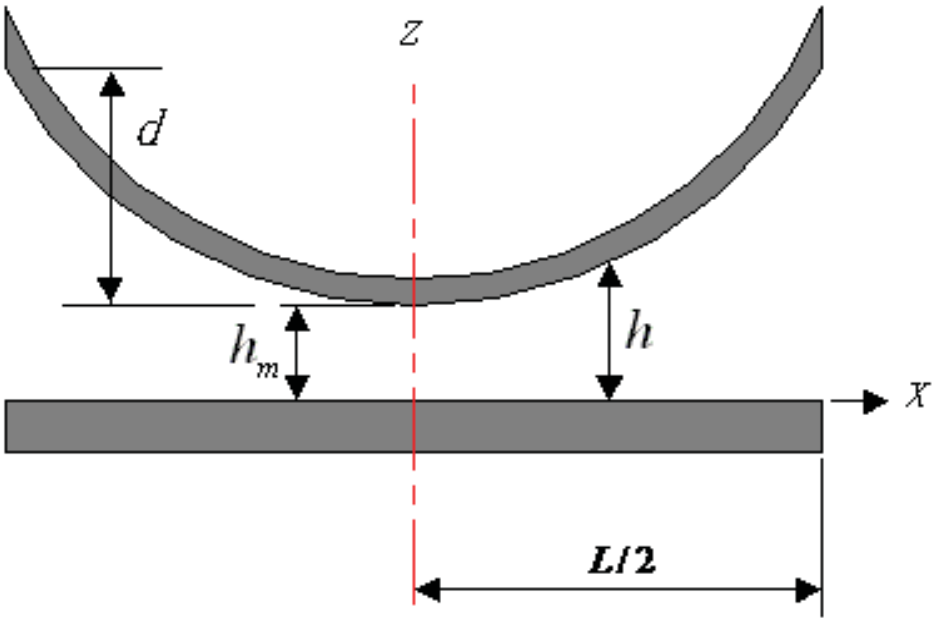

Figure 1 Squeeze film geometry between convex curved plates with a cosine form

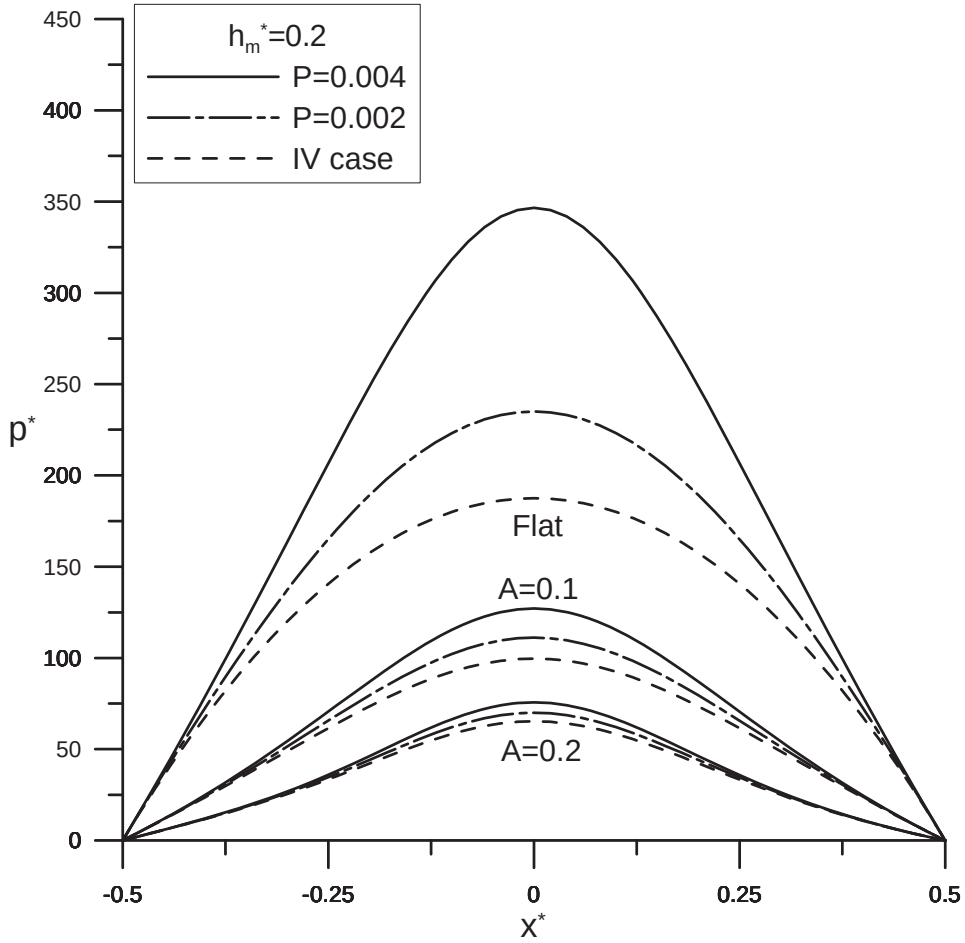

Figure 2 Squeeze film pressure p^* as a function of the horizontal coordinate x^* for different values of the amplitude ratio A and the PDV parameter P

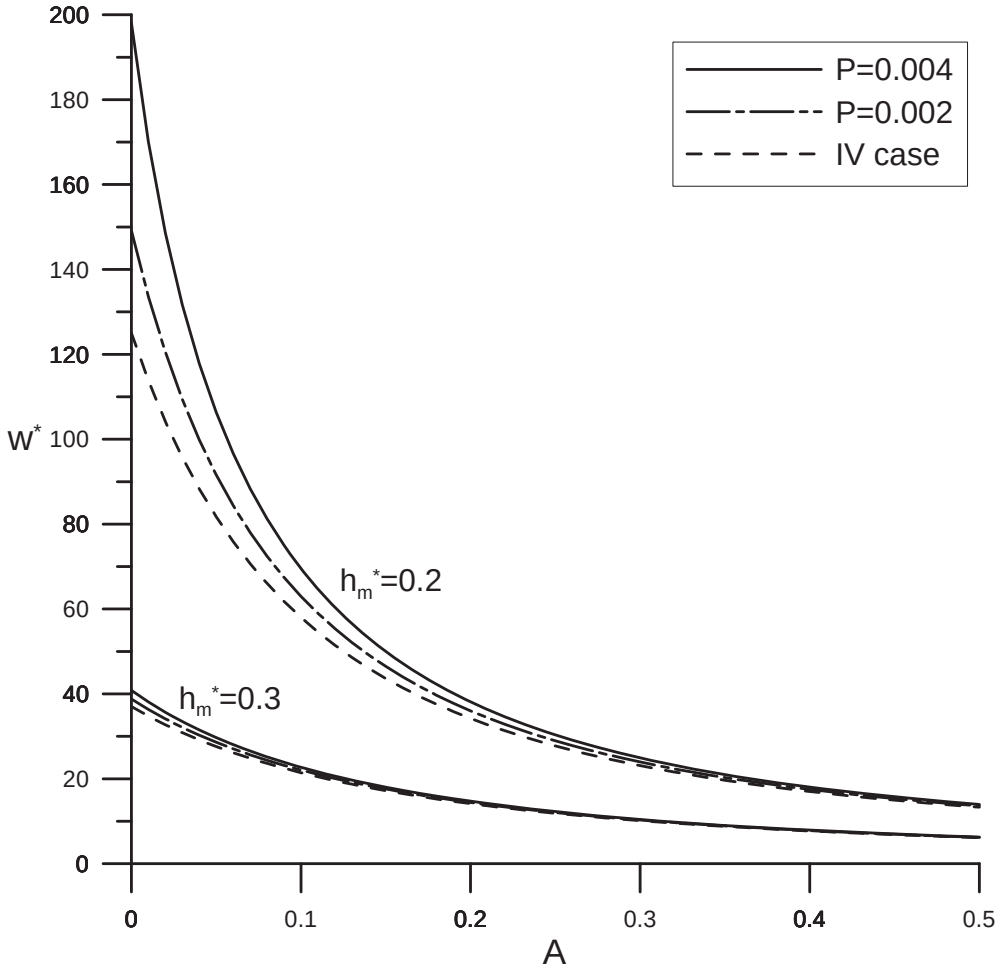

Figure 3 Load-carrying capacity W^* as a function of the amplitude ratio A for different values of the minimum film thickness h_m^* and the PDV parameter P

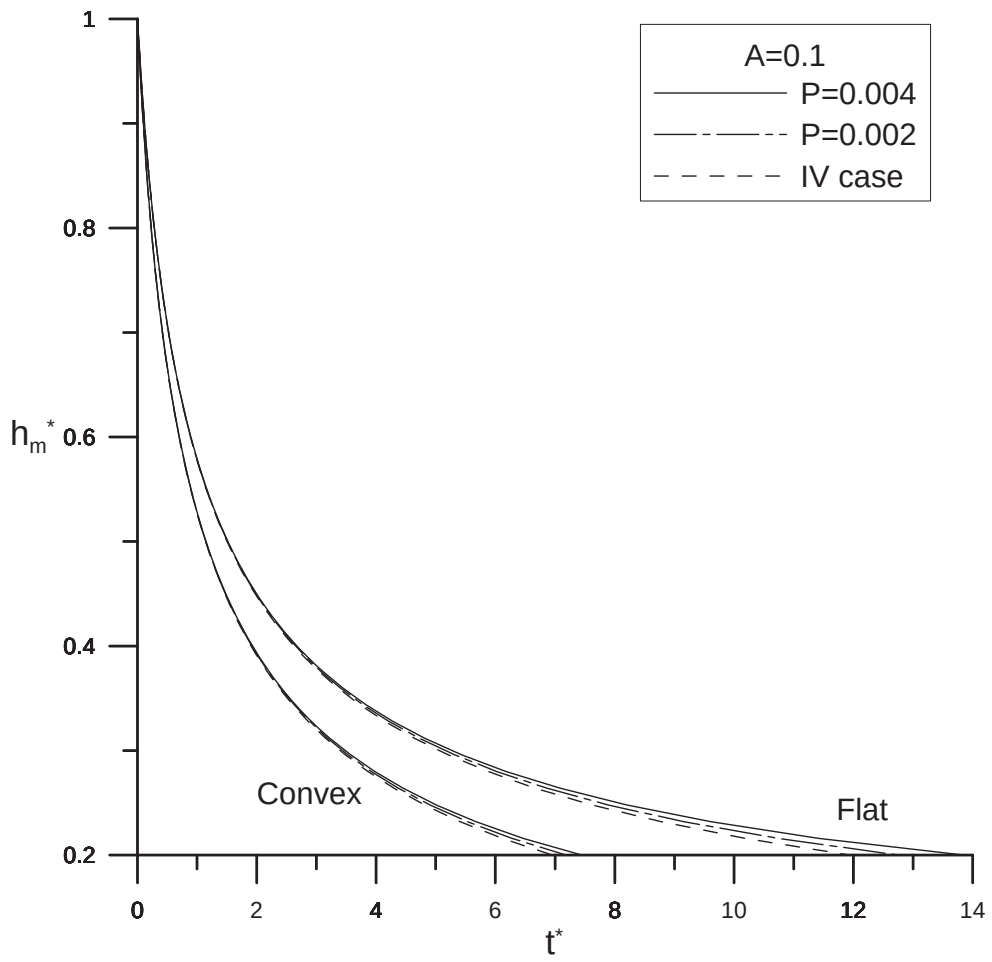

Figure 4 Minimum film height h_m^* as a function of the elapsed time t^* for different values of the PDV parameter P under the amplitude ratio $A = 0.1$