ANALYSIS OF SQUEEZE FILM CHARACTERISTICS BETWEEN TWO CYLINDERS: NON-NEWTONIAN COUPLE STRESS FLUID MODEL

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Abstract

Grounded on the Stokes micro-continuum theory, the effects of non-Newtonian couple stresses upon the squeeze film behavior between two cylinders are theoretically investigated. The modified Reynolds-type equation governing the squeeze film pressure is derived to account for the couple stress effects resulting from the lubricant blended with various additives. Comparing with the conventional Newtonian-lubricant case, the couple stress effects signify an improvement in the squeeze film characteristics. Increasing the couple stress parameter increases the load-carrying capacity. By the use of couple stress fluids as the lubricant, the squeeze film system provides a longer time to prevent cylinder-cylinder contact and results in longer bearing life.

Key Words: micro-continuum theory, non-Newtonian couple stress fluids, squeeze films, cylinder-cylinder system

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兩圓柱體間擠壓膜性能之分析:

非牛頓偶應力流體模式

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摘要

在史托克微連體理論之基礎下，本文探討非牛頓偶應力效應對於兩圓柱體間擠壓膜性能之影響。在分析與推導過程中，本研究得到一條非牛頓雷諾型方程式，可用以涵蓋傳統牛頓流體潤滑劑在加入各種不同添加劑後所生之偶應力效應。根據所得結果，若與使用傳統牛頓流體潤滑劑之情況比較，則非牛頓偶應力效應可以改善兩圓柱體間之擠壓膜性能。當擠壓膜系統使用非牛頓偶應力流體為潤滑劑，則可提高兩圓柱體間擠壓行為之負荷能力，同時亦可延長擠壓機構之操作生命。

關鍵詞: 微連體理論，非牛頓偶應力流體，擠壓膜，圓柱-圓柱系統

1. INTRODUCTION

Application of squeeze film mechanisms is extensively observed in engineering science such as matching gears, machine tools, dampers and human joints. Conventionally, analyses of squeeze film characteristics for various types of mechanisms focus upon the use of Newtonian viscous fluids by Pinkus and Sternlicht [1] and Hamrock [2]. Because of the developing of modern machine elements, the increasing use of fluids blended with various types of additives has been emphasized. These kinds of complex fluids exhibit non-Newtonian flow behaviors. In order to describe the flow characteristics of these kinds of non-Newtonian fluids, a micro-continuum theory of couple stress fluids have been proposed by Stokes [3]. The Stokes micro-continuum theory defines the rotational field in terms of the velocity field for setting up the constitutive relationship between stress and strain rate, and allows for polar effects such as the presence of couple stresses, body couples and a non-symmetric stress tensor. Application of this couple stress fluid model to engineering sciences, many articles regarding the squeeze film problems have been presented. Representative studies are observed in the squeeze film plates of various shapes by Ramanaiah [4], the squeeze-film configuration with reference to synovial joints by Bujurke and Jayaraman [5] and Lin [6], the squeeze-film partial journal bearings by Lin [7], the squeeze film behavior between a sphere and a flat plate [8] and the parallel circular squeeze-film plates [9]. However, we have no idea how the couple stress effects affect the cylinder-cylinder system. A further study is therefore motivated.

On the ground of the Stokes micro-continuum theory, it is mainly concerned with the non-Newtonian effects of couple stresses upon the squeeze film performance between two cylinders. The modified Reynolds-type equation governing the squeeze film pressure is derived using the Stokes motion equations to account for the lubricant blended with various types of additives. Comparing with the conventional case of Newtonian lubricant, the squeeze film characteristics including the film pressure, the load-carrying capacity and time-height relationship are presented for various values of couple stress parameter.

2. ANALYSIS

The squeeze film geometry between two long cylinders with the same radius R is considered in Figure 1. The upper rigid cylinder is approaching the lower rigid

cylinder with a squeezing velocity $V = -\partial h / \partial t$. The lubricant in the film region is taken to be a Stokes couple stress fluid. Under the assumption that the fluid film is thin, the fluid inertia is small, and the body forces and body couples are absent, the continuity equation and the momentum equations from Stokes [3] reduce to

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \tag{1}$$

$$\frac{\partial p}{\partial x} = \mu \frac{\partial^2 u}{\partial z^2} - \eta \frac{\partial^4 u}{\partial z^4} \tag{2}$$

$$\frac{\partial p}{\partial z} = 0 \tag{3}$$

The boundary conditions for the velocity component $u(x,z)$ are:

$$u\big|_{z=-h_1} = \frac{\partial^2 u}{\partial z^2}\bigg|_{z=-h_1} = 0 \tag{4}$$

$$u\big|_{z=+h_2} = \frac{\partial^2 u}{\partial z^2}\bigg|_{z=h_2} = 0 \tag{5}$$

The total film thickness h, provided $R >> r$, is approximated by Hamrock [2].

$$h = h_1 + h_2 = h_m + \frac{x^2}{R} \tag{6}$$

$$h_1 = h_2 = \frac{h_m}{2} + \frac{x^2}{2R} \tag{7}$$

where h_1 is the upper film thickness, h_2 is the lower film thickness and h_m denotes the minimum film thickness. Solving equation (2) with the above boundary conditions one can obtain the expression of u.

$$u = \frac{1}{2\mu}\frac{dp}{dx}\left\{z^2 - \frac{1}{4}h^2 + 2l^2\left\{1 - \frac{\sinh[(2z+h)/(2l)] - \sinh[(2z-h)/(2l)]}{\sinh(h/l)}\right\}\right\} \tag{8}$$

where

$$l = (\eta / \mu)^{1/2} \tag{9}$$

Now integrate the continuity equation (1) with respect to z with the boundary conditions of w:

$$w\big|_{z=-h_1} = 0 \qquad \text{and} \qquad w\big|_{z=h_2} = \frac{\partial h}{\partial t} \tag{10}$$

The modified Reynolds-type equation governing the film pressure is obtained.

$$\frac{\partial}{\partial x}\left\{\left[h^3 - 12l^2 h + 24l^3 \tanh\left(\frac{h}{2l}\right)\right]\frac{dp}{dx}\right\} = 12\mu\frac{\partial h}{\partial t} \tag{11}$$

To proceed the analysis, one introduces the dimensionless variables and parameters:

$$X = \frac{x}{R}, \qquad P = \frac{ph_{m0}^2}{\mu R(-\partial h/\partial t)}, \qquad H_m = \frac{h_m}{h_{m0}},$$
$$\tau = \frac{l}{h_{m0}}, \qquad \gamma = \frac{h_{m0}}{R} \tag{12}$$

where h_{m0} represents the initial minimum film thickness. The dimensionless modified Reynolds-type equation is then expressed as the following.

$$\frac{\partial}{\partial X}\left[F(H,\tau)\frac{dP}{dX}\right] = -\frac{12}{\gamma} \tag{13}$$

where the functions H and F are defined by:

$$H = \frac{h}{h_{m0}} = H_m + \frac{X^2}{\gamma} \tag{14}$$

$$F(H,\tau) = H^3 - 12\tau^2 H + 24\tau^3 \tanh\left(\frac{H}{2\tau}\right) \tag{15}$$

The boundary conditions for the film pressure are:

$$P = 0 \qquad \text{at} \qquad X = \pm 1 \tag{16}$$

$$\frac{dP}{dX} = 0 \qquad \text{at} \qquad X = 0 \tag{17}$$

Integrating the Reynolds equation with respect to X with the above conditions, the squeeze film pressure is obtained.

$$P = \frac{12}{\gamma}\int_X^{X=1} \frac{X}{F(H,X)}dX \tag{18}$$

3. SQUEEZE FILM CHARACTERISTICS

The squeeze film pressure presented in (18) cannot be calculated by direct integration, it could be numerically evaluated by the method of Gaussian quadrature. When the film pressure is known, the squeeze film characteristics including the load-carrying capacity and the approaching time can be obtained. By integrating the film pressure acting upon the upper cylinder one can obtain the load-carrying capacity.

$$L = \int_{x=-R}^{R} pBdx \tag{19}$$

where B denotes the length of the cylinder. Use in a dimensionless form:

$$W = \frac{Lh_{m0}^2}{\mu R^2 B(-\partial h/\partial t)} = \frac{12}{\gamma}\int_{X=-1}^{1} \frac{X^2}{F(H,\tau)}dX \tag{20}$$

Let the dimensionless approaching time be:

$$T = \frac{Lh_{m0}^2}{\mu R^3 B} t \tag{21}$$

The time-height relationship can be obtained from (20).

$$\frac{dH_m}{dT} = -\frac{1}{12\int_{X=-1}^{1} [X^2 / F(H,\tau)] dX} \tag{22}$$

The initial condition is:

$$H_m = 1 \quad \text{at} \quad T = 0 \tag{23}$$

Using the fourth-order Runge-Kutta method, one can obtain a numerical solution of the minimum film height depending the approaching time.

4. RESULTS AND DISCUSSION

Grounded on the Stokes micro-continuum theory, the non-Newtonian effects of couple stresses upon the squeeze film characteristics between two rigid cylinders are investigated. To account for the couple stress effects arising from a Newtonian lubricant blended with various types of additives, the modified Reynolds-type equation is derived by using the Stokes motion equations. According to the Stokes theory of couple stress fluids, the new material constant η in equation (2) is responsible for the property of couple stresses. Since the dimension of l in equation (9) is of length, it can be considered as the characteristic length of additives in a Newtonian lubricant. Therefore, the non-Newtonian effects of couple stresses are characterized by the couple stress parameter τ defined in equation (12). When the value of τ is zero, the dimensionless modified Reynolds-type equation reduces to the conventional Newtonian-lubricant case. To obtain the results, the squeeze film characteristics are presented for the parameters with $\gamma = 0.05$ and $\tau = 0 \sim 0.5$. Figure 2 presents the dimensionless film pressure P generated by the squeeze film action as a function of dimensionless coordinate X for different values of couple stress parameter τ at dimensionless minimum film height $H_m = 0.4$. It is observed that the couple stress effects ($\tau = 0.1$, 0.2 and 0.3) result in a higher film pressure than the Newtonian-lubricant case, especially in the vicinity of the position of minimum film height. Figure 3 describes the dimensionless maximum film pressure P_M as a function of dimensionless minimum film height H_m for different values of couple stress parameter τ. Compared with the Newtonian-lubricant case, the effects of couple stresses signify an increase in the value of P_M. Moreover, the lower the minimum squeeze film height is, the more the couple stresses affect the maximum

film pressure.

Figure 4 shows the dimensionless load-carrying capacity W versus dimensionless minimum film height H_m for different values of couple stress parameter τ. Since the effects of couple stresses yield a higher film pressure, the integrated load-carrying capacity is similarly affected. Compared with the Newtonian-lubricant case, the couple stress effects ($\tau = 0.1$, 0.2 and 0.3) are observed to increase the load-carrying capacity. Figure 5 describes the dimensionless load-carrying capacity W versus couple stress parameter τ at various dimensionless minimum film heights H_m. As shown, the effects of couple stresses provide an increase in the load-carrying capacity. Moreover, a larger increment of the load is obtained with an increasing value of τ or a decreasing value of H_m.

Figure 6 presents the dimensionless approaching time T versus dimensionless minimum film height H_m for different values of couple stress parameter τ. The effects of couple stresses ($\tau = 0.2$, 0.3, 0.4 and 0.5) are observed to provide an increase in the approaching time of upper cylinder. Figure 7 shows the dimensionless approaching time T versus couple stress parameter τ at dimensionless minimum film heights H_m. As seen, increasing the value of τ lengthens the approaching time T. These phenomena can be realized that since the couple stress effects result in a higher load-carrying capacity, a higher H_m would be attained for the same time to be taken as compared to the Newtonian-lubricant case. In other words, the time of approach for the cylinder-cylinder system is lengthened by the use of an incompressible couple stress fluid.

5. CONCLUSIONS

On the ground of Stokes micro-continuum theory, the effects of couple stresses on the squeeze film motion between two rigid cylinders are presented. The modified Reynolds-type equation governing the squeeze film pressure is derived to account for the couple stress effects resulting from a Newtonian lubricant blended with various types of additives. In accordance with the results obtained, the squeeze film characteristics of the cylinder-cylinder system are significantly affected by the presence of couple stresses and dominated by the couple stress parameter. Comparing with the conventional Newtonian-lubricant case, the effects of couple stresses increase the load-carrying capacity and lengthen the time of approach. Furthermore, the quantitative effects are more pronounced for the system operating at small values of the squeezing height.

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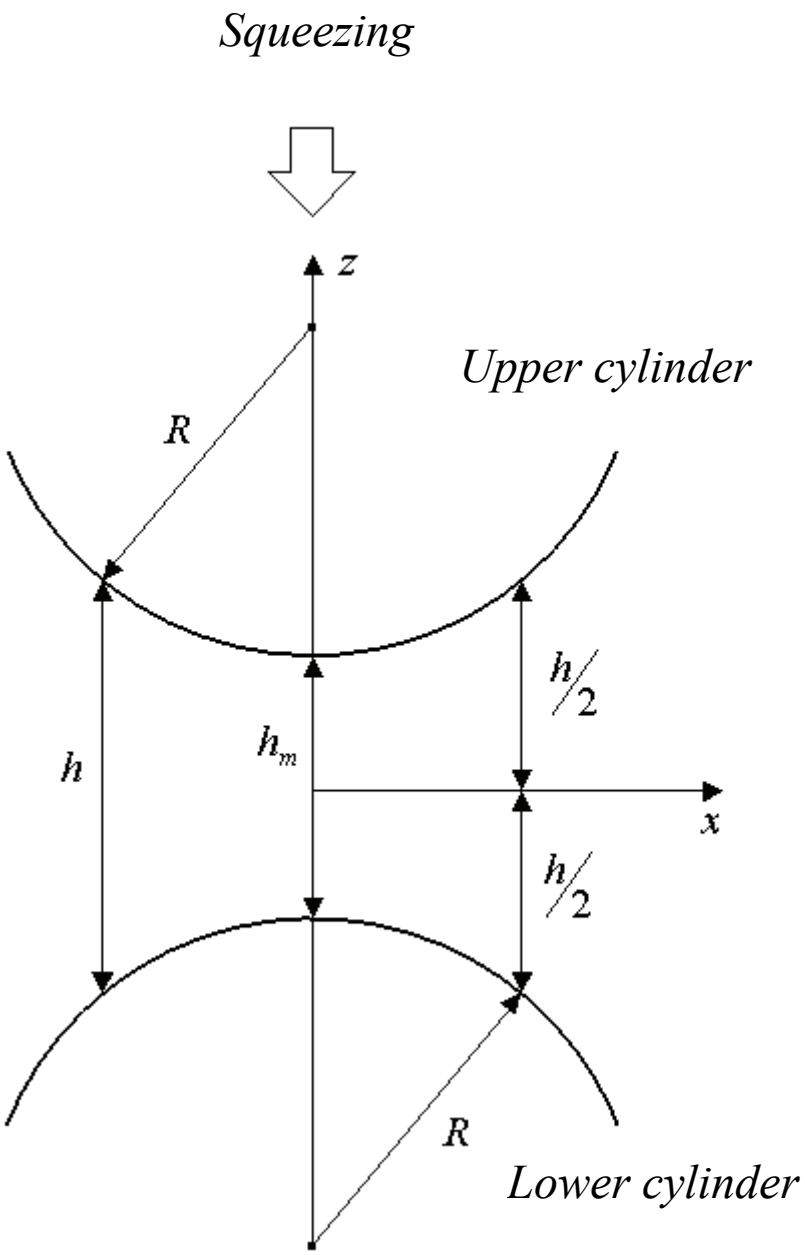

Figure 1 Squeeze film geometry between two cylinders.

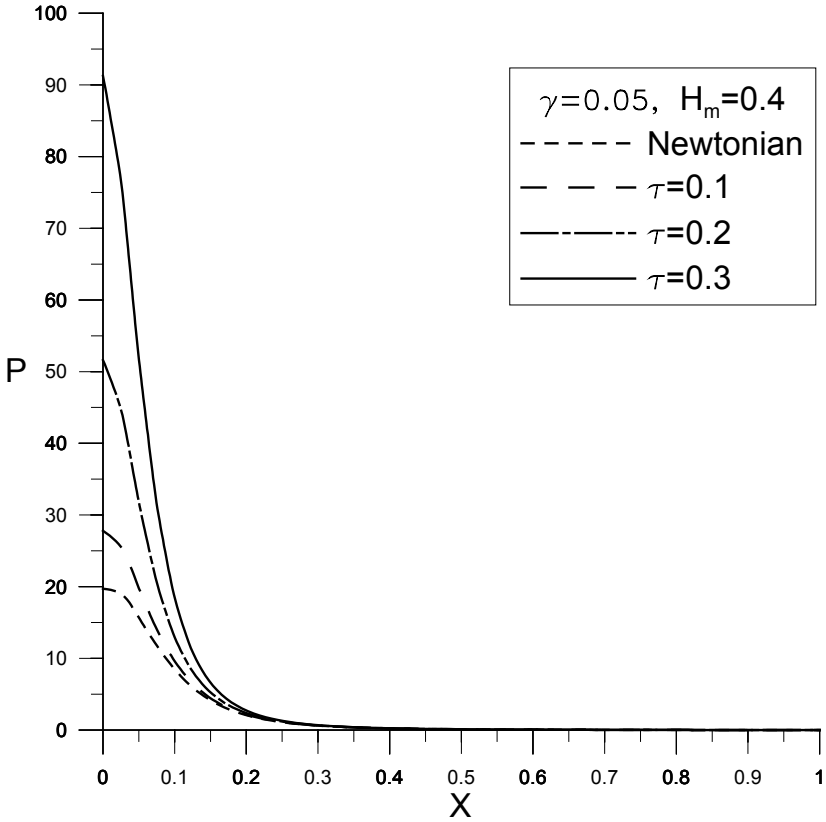

Figure 2 Dimensionless film pressure P as a function of X for different τ .

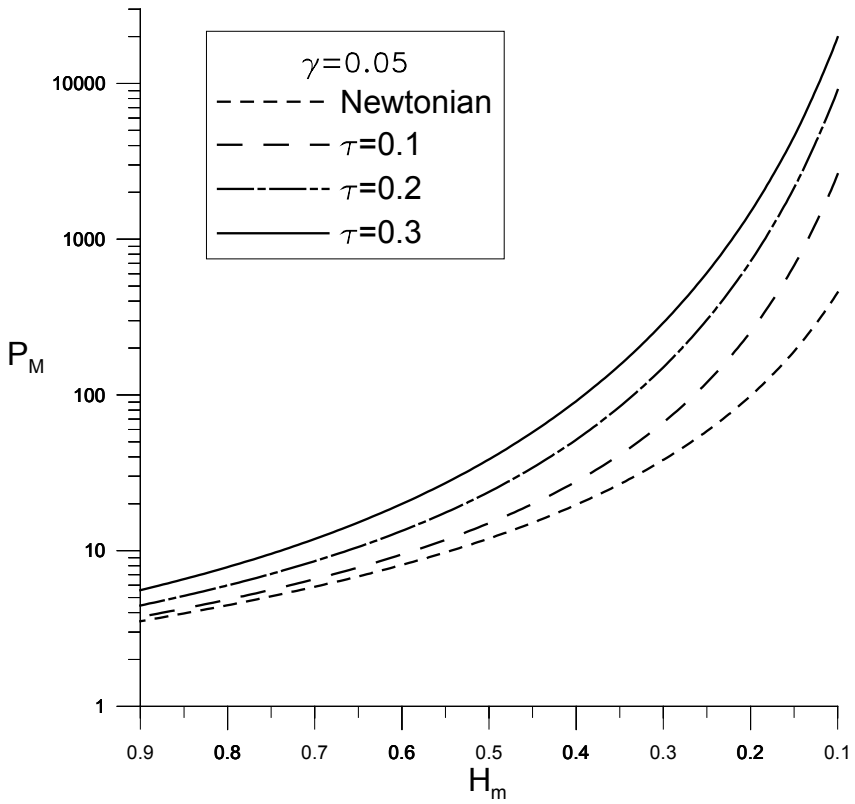

Figure 3 Dimensionless maximum pressure P_M as a function of H_m for different τ .

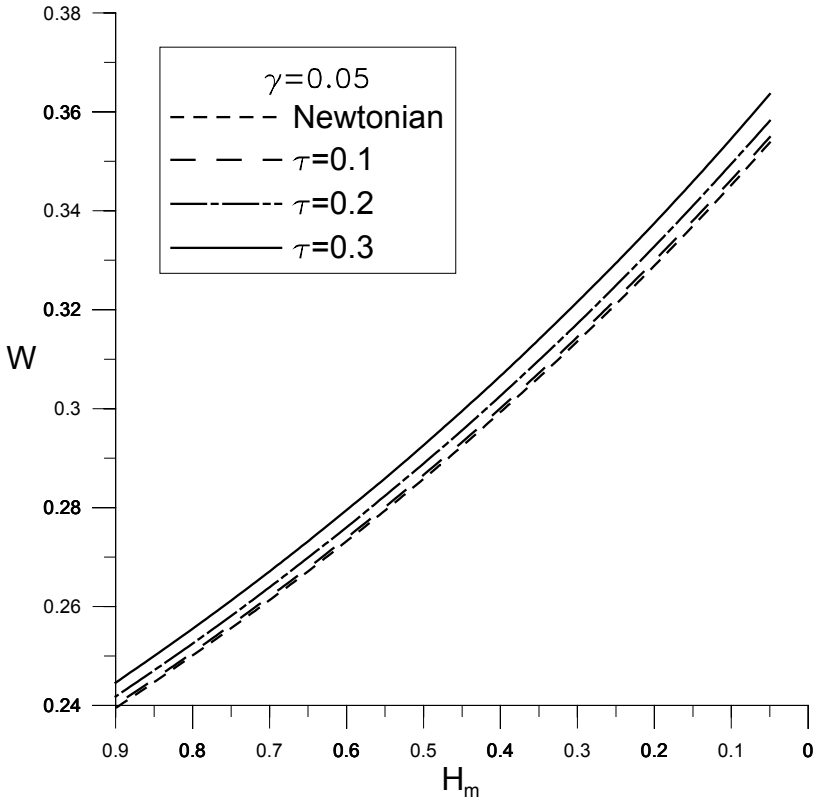

Figure 4 Dimensionless load-carrying capacity W versus of H_m for different τ .

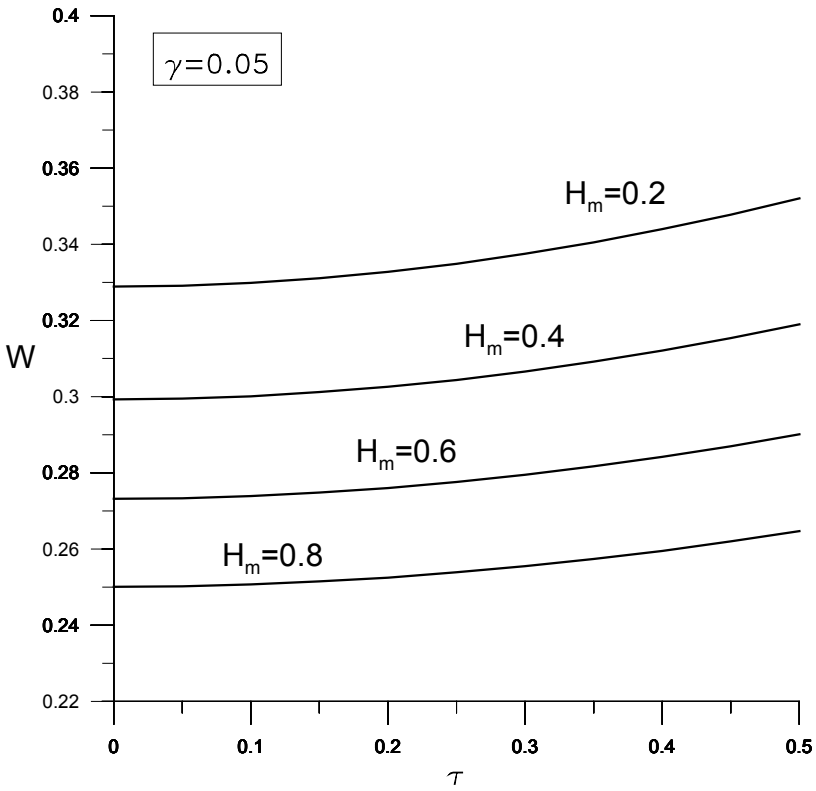

Figure 5 Dimensionless load-carrying capacity W^* versus of τ for different H_m.

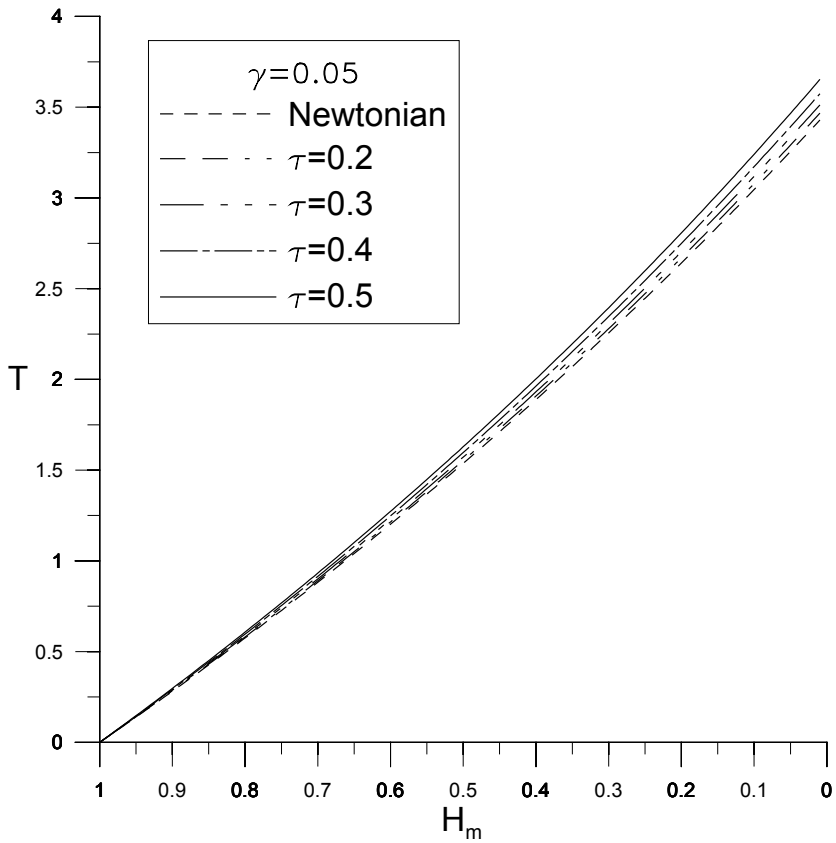

Figure 6 Dimensionless approaching time T versus H_m for different τ .

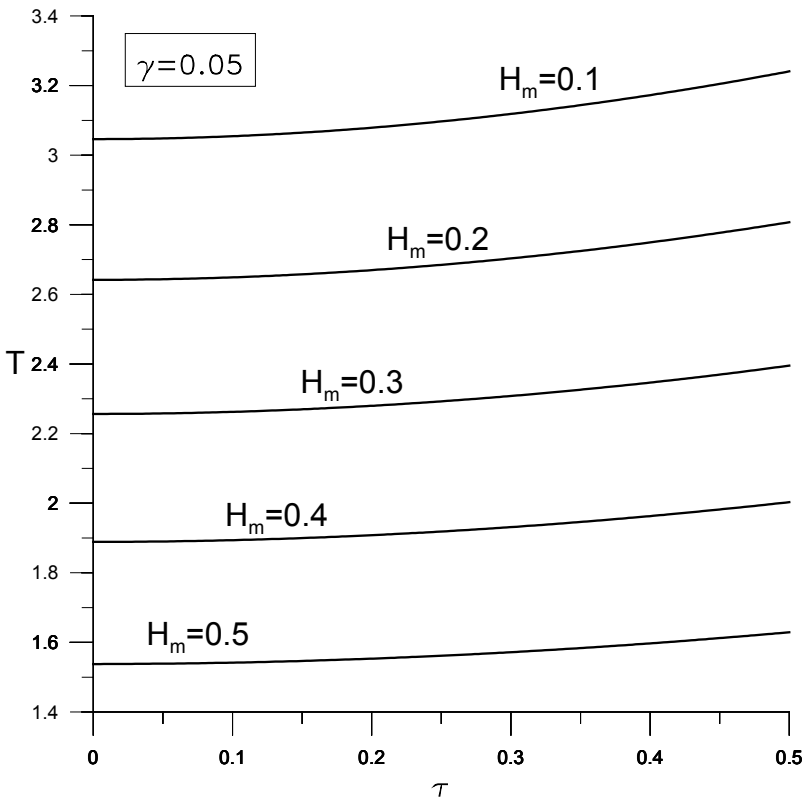

Figure 7 Dimensionless approaching time T versus τ at different H_m.