A FUZZY MULTIPLE ATTRIBUTES DECISION-MAKING FOR EVALUATING FLEXIBILITY IN A MANUFACTURING SYSTEM

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Abstract

Evaluation of the flexibility in a manufacturing system (manufacturing flexibility) is very important to determining the competitiveness of manufacturing system, and is being increasing discussed in the literature on manufacturing system. Traditional approaches have tried objectively to evaluate manufacturing flexibility, and are theoretical and involve only two or three dimensions. The importance grade of flexibility dimensions, and the subjective evaluation of manufacturing flexibility have seldom been addressed. The objective of this paper is to develop a multiple attributes decision-making model with two-stage assessment to the evaluation of the manufacturing flexibility in a manufacturing system development. The evaluation problem is solved by a fuzzy fusion method based on the maximum entropy ordered weighted averaging operators. We also provided a fuzzy fusion method of linguistic information. While evaluating the degree of manufacturing flexibility, one may find the need for improving manufacturing flexibility, and determine the dimensions of manufacturing flexibility as the best directions to improvement until she/he can accept it. An example is used to demonstrate the proposed method. Finally, we also show that the linguistic assessment of manufacturing flexibility is reasonable.

Keywords: Manufacturing flexibility; Multiple attributes analysis; maximum entropy ordered weighted averaging operators; Two-stage assessment

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模糊多屬性決策方法應用於製造彈性評估之研究

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摘要

對於確保製造系統的競爭力，製造彈性評估具有重要的策略性功能，在相關文獻的討論日益增加。針對製造系統的彈性評估，傳統衡量方法都是採取客觀屬性，以確定數值來衡量兩個或三個彈性維度，並且在評估製造彈性時大多沒有考慮彈性維度的重要性與主觀性評估。本文提出一種多屬性決策模式，其中運用 MEOWA 運算子的語意融合方法，適合於不確定決策環境下製造彈性評估。在評估製造彈性時，不僅可以發現製造彈性的改善時機，更可以確定製造彈性的最佳改善方向，直到評估者滿意為止。最後，例釋與模擬本方法，顯示出本法之語意評估是合理的。

關鍵字：製造彈性，多屬性分析，MEOWA 運算子，兩階段評估

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1. Introduction

Manufacturing environments have changed so fast in recent decades that the flexibility of manufacturing systems has become increasingly important. Flexible manufacturing systems, computer-integrate manufacturing systems, Just-in-Time systems, flexible factories, and so forth, are rely on manufacturing flexibility. Generally, manufacturing flexibility (MF) is the ability of a manufacturing system to cope with environmental changes effectively and efficiently. MF has been emphasized as a major competitive priority in manufacturing system (Beach *et al.*, 2000; Sethi and Sethi, 1990). Flexibility improvement is an important issue on the operations managers that must be evaluating the degree of MF when making capital investment decisions and measuring performance (Gerwin, 1993). However, MF is a complex, multidimensional and difficult-to-synthesize concept (Sethi and Sethi, 1990), and the needs of operations managers have not yet been met (De Toni and Tonchia, 1998; Gupta and Goyal, 1989; Sarker *et al.*, 1994; Vokurka and O'Leary-Kelly, 2000).

Many researchers have considered definitions, requests, classificatory in dimensions, measurement, choices, and interpretations of MF (Gupta and Goyal, 1989; Sethi and Sethi, 1990; Sarker *et al.*, 1994; De Toni and Tonchia, 1998; Beach *et al.*, 2000; Vokurka and O'Leary-Kelly, 2000). Upton (1994) proposed a framework for analyzing MF according to different dimensions, each of which to cope with the environmental changes at different time intervals and is specified by three elements: range, mobility and uniformity. Golden and Powell (2000) presented an inclusive definition in which flexibility can be measured by four metrics: efficiency, responsiveness, versatility and robustness. Many researchers have tried objectively to evaluate MF. Several efforts are theoretical and involve only two or three dimensions (Gupta and Goyal, 1989; Beach *et al.*, 2000). The importance grade for flexibility dimensions, and the subjective evaluation of MF have seldom been addressed.

Most operations managers cannot give exact numerical values to represent opinions, based on human perception, on flexibility metrics, more realistic evaluation uses linguistic assessments rather than numerical values (Beach *et al.*, 2000; Gerwin, 1993; Herrera *et al.*, 2000; Vokurka and O'Leary-Kelly, 2000). In fact the flexibility metrics are specified as linguistic terms, such as very high, high, middle, low, and very low. After Zadeh (1965) introduced fuzzy set theory to deal with vague problems, linguistic terms

have been used for approximate reasoning within the framework of fuzzy set theory to handle the ambiguity of evaluating data and the vagueness of linguistic expression (Zadeh, 1975). Normal trapezoidal fuzzy numbers have been used to characterize linguistic terms used in approximate reasoning.

Therefore, the purpose of this paper is to construct a fuzzy multiple attributes decision-making model with linguistic information for MF evaluation problem. In the fuzzy linguistic assessment, according to the uncertainty of evaluation information, a linguistic quantifier chosen by decision makers will be used in maximum entropy ordered weighted averaging (MEOWA) operators as an effective alternative for evaluating the decision makers' optimism. An algorithm is proposed to determine the degree of MF in a fuzzy environment, and we also developed a fuzzy fusion method of linguistic information. Section 2 presents a fuzzy fusion method. Section 3 presents a hierarchical structure model of flexibility in a manufacturing system development. Section 4 describes a two-stage assessment and an example for determining the degree of MF. Finally, the reasonability of the degree of MF is discussed.

2. Fusion of Linguistic Information

The fuzzy linguistic approach assesses the linguistic variables using words or sentences in natural language (Zadeh, 1975). This approach is appropriate for some problems in which information may be qualitative, or quantitative information may not be stated precisely, since either it is unavailable or the cost of its computation is prohibitive, such that an 'approximate value' suffices (Herrera and Herrera-Viedma, 2000). In applying a fuzzy linguistic approach to the measurement of manufacturing flexibility, only the performance of flexibility metrics are classified into low, middle or high and neglect the important of the flexibility that will induce the imprecision and bias. Therefore, importance for each of flexibility metric should be evaluating to get the degree of MF in a manufacturing system.

As mentioned above, the performance rating and importance grade should be rated for each item. Consequently, both were scored on a seven-rank scale, as shown in Table 1. The seven linguistic terms in $S=\{s_0, s_1,\cdots, s_6\}$ were specified by the linguistic value 1, 2, $\cdots$, 7. The semantics of each term s_i are expressed as a trapezoidal membership functions associated with a normal fuzzy number, as listed in Table 1. Not all individuals

agree on the same associations between membership functions and linguistic scale. However, this paper considers a situation in which managers can perfectly distinguish a set of linguistic terms, and can use linguistic terms to express their opinions.

The criteria ratings of flexibility are linguistic variables with 11 values, which are treated as fuzzy numbers with trapezoidal membership functions, as shown in Table 2. The defuzzification by the centroid method is defined as:

$$\int_a^b x\,\mu(x)dx / \int_a^b \mu(x)dx, \qquad (1)$$

where *a* and *b* are lower and upper limits of the integral, respectively. This work has its centroid

Table 1. Linguistic scale and fuzzy numbers of performance rating and importance grade

Seven ranks of performance rating and importance grade	Fuzzy number
1：s_0 ：Definitely low（DL）	s_0 =（0, 0, 0, 0.1）
2：s_1 ：Very low（VL）	s_1 =（0, 0.1, 0.2, 0.3）
3：s_2 ：Low（L）	s_2 =（0.2,0.3,0.4,0.5）
4：s_3 ：Middle（M）	s_3 =（0.4,0.5,0.5,0.6）
5：s_4 ：High（H）	s_4 =（0.5,0.6,0.7,0.8）
6：s_5 ：Very high（VH）	s_5 =（0.7,0.8,0.9,1.0）
7：s_6 ：Definitely high（DH）	s_6 =（0.9,1.0,1.0,1.0）

Table 2. Linguistic scale and fuzzy numbers of criteria rating of flexibility

Eleven ranks of criteria rating	Fuzzy number
1：v_0：Definitely low	v_0=（0.0, 0.0, 0.0, 0.1）
2：v_1：Extra low	v_1=（0.0, 0.1, 0.1, 0.2）
3：v_2：Very low	v_2=（0.1, 0.2, 0.2, 0.3）
4：v_3：Low	v_3=（0.2, 0.3, 0.3, 0.4）
5：v_4：Slightly low	v_4=（0.3, 0.4, 0.4, 0.5）
6：v_5：Middle	v_5=（0.4, 0.5, 0.5, 0.6）
7：v_6：Slightly high	v_6=（0.5, 0.6, 0.6, 0.7）
8：v_7：High	v_7=（0.6, 0.7, 0.7, 0.8）
9：v_8：Very high	v_8=（0.7, 0.8, 0.8, 0.9）
0：v_9：Extra high	v_9=（0.8, 0.9, 0.9, 1.0）
11：v_{10}：Definitely high	v_{10}=（0.9, 1.0, 1.0, 1.0）

$G(v_0)=0.0333$, $G(v_1)=0.1$, $G(v_2)=0.2$, $G(v_3)=0.3$, $G(v_4)=0.4$, $G(v_5)=0.5$, $G(v_6)=0.6$, $G(v_7)=0.7$, $G(v_8)=0.8$, $G(v_9)=0.9$, $G(v_{10})=0.9667$ as center of mass of v_0, v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, respectively.

Let $V=\{v_0, v_1, \cdots, v_{10}\}$ be the set of the criteria rating of flexibility for each item, and $S=\{s_0, s_1, \cdots, s_6\}$ with 11 and 7 terms, respectively. By the transformation function defined by Herrera *et al.*[8], the linguistic term of S can be transformed into a unified linguistic term set, V, that a fuzzy assessment matrix for S×V can be formed. The transformation function, θ_{SV} is defined as:

$\theta_{SV}: S \rightarrow F(V)$,

$\theta_{SV}(s_i)=\{(v_j, u_{ij}) / j \in \{0, 1, \cdots, 10\}\}$, for $s_i \in S$, (2)

$u_{ij} = \max \min\{\mu_{si}(x), \mu_{vj}(x)\}$,

where F(V) is the set of fuzzy sets defined in V, and $\mu_{si}(x)$ and $\mu_{vj}(x)$ are the membership functions of the fuzzy sets associated to the terms s_i and v_j, respectively. Therefore, the result of θ_{SV} for any linguistic term of S is a fuzzy set defined in V as follow:

$\theta_{SV}(s_0)=\{(v_0,1),(v_1,0.5),(v_2,0),(v_3,0),(v_4,0),(v_5,0),(v_6,0),(v_7,0),(v_8,0),(v_9,0),(v_{10},0)\}$,

$\theta_{SV}(s_1)=\{(v_0,0.5),(v_1,1),(v_2,1),(v_3,0.5),(v_4,0),(v_5,0),(v_6,0),(v_7,0),(v_8,0),(v_9,0),(v_{10},0)\}$,

$\theta_{SV}(s_2)=\{(v_0,0),(v_1,0),(v_2,0.5),(v_3,1),(v_4,1),(v_5,0.5),(v_6,0),(v_7,0),(v_8,0),(v_9,0),(v_{10},0)\}$,

$\theta_{SV}(s_3)=\{(v_0,0),(v_1,0),(v_2,0),(v_3,0),(v_4,0.5),(v_5,1),(v_6,0.5),(v_7,0),(v_8,0),(v_9,0),(v_{10},0)\}$,

$\theta_{SV}(s_4)=\{(v_0,0),(v_1,0),(v_2,0),(v_3,0),(v_4,0),(v_5,0.5),(v_6,1),(v_7,1),(v_8,0.5),(v_9,0),(v_{10},0)\}$,

$\theta_{SV}(s_5)=\{(v_0,0),(v_1,0),(v_2,0),(v_3,0),(v_4,0),(v_5,0),(v_6,0),(v_7,0.5),(v_8,1),(v_9,1),(v_{10},0.5)\}$,

$\theta_{SV}(s_6)=\{(v_0,0),(v_1,0),(v_2,0),(v_3,0),(v_4,0),(v_5,0),(v_6,0),(v_7,0),(v_8,0),(v_9,0.5),(v_{10},1)\}$.

With respect to fusion of linguistic information, this paper uses a MEOWA weighing vector, $W^* = (w_1^*, w_2^*, \cdots, w_n^*)$, as follows. Let $\{a_1, a_2, \cdots, a_n\}$ be a set of numerical values to be aggregate, then the MEOWA operator Φ_Q with the linguistic quantifier Q, is defined as:

$$\Phi_Q(a_1, a_2, \cdots, a_n) = W^* \cdot B^T = \sum_{j=1}^{n} w_j^* b_j \ , \tag{3}$$

where $W^* = (w_1^*, w_2^*, \cdots, w_n^*)$, is a MEOWA weighting vector, such that, $w_i^* \in [0, 1]$ and $\sum_i w_i^* = 1$ (see the appendix), $B = (b_1, b_2, \cdots, b_n)$, is the associated ordered value vector, each element $b_i \in B$ is the ith largest value in the $\{a_1, a_2, \cdots, a_n\}$,

Yager (1988) introduced the OWA operators, which used in the problem of multiple attribute decision-making, to provide a family of aggregation operators that always lies between the 'and' and the 'or'. O'Hagan (1988) developed a method to obtaining the maximum

entropy OWA (MEOWA) weighting vector that have a predefined degree of orness and that maximize the entropy. This approach is based upon the solution of a constraint optimization problem. Filev and Yager (1995) suggested a two-step process used for obtain the MEOWA weighting vector that generate some prescribed degree of orness without having to solve the constraint optimization problem. Mitchell and Estrakh (1997) presented an application of the MEOWA operator to lossless image compression. An algorithm for calculating the MEOWA weighting vector is presented in Appendix, and can be computerized by a computer program written in Matlab language.

3. Hierarchical Structure Model of Manufacturing Flexibility

A systematic approach is proposed to evaluate the degree of MF, using a fuzzy set theory and hierarchical structure analysis. This method is suitable for decision-making in a fuzzy environment. The dimensions of flexibility proposed by Gerwin (1993), Slack (1987) were expressed in seven dimensions as mix, changeover, modification, volume, rerouting, material and sequencing flexibility, etc., based on the relationship between MF and environmental changes. Furthermore, each dimension was divided into

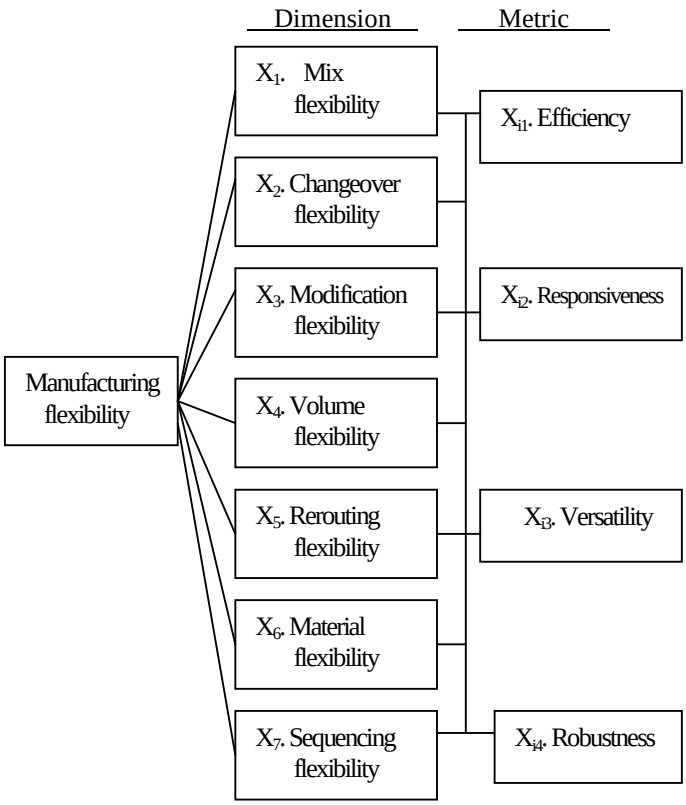

Figure 1. Hierarchical structure model of manufacturing flexibility

four flexibility metrics (Golden and Powell, 2000), such as efficiency, responsiveness, versatility and robustness. For convenience, the dimension mix flexibility was represented as X_1, changeover flexibility as X_2, and so on. The metrics of the ith flexibility dimension were represented as in Figure 1; for example, efficiency was denoted by X_{i1}, responsiveness by X_{i2}, and so on.

The decision-makers consider the importance grade and related performance rating, grading both as $S=\{s_0, s_1,\cdots, s_6\}$. Suppose the degree of MF is to be assessed by operations managers, such as the vice president of manufacturing, plant manager or management consultant, and so on, whose collective experience extends across a broad range of manufacturing environment and its environmental changes. The symbol I_i is used to denote the importance grade of dimension X_i; P_{ij} performance rating of flexibility metric X_{ij}, according to the operations manager's assessing data ($i = 1, 2, \cdots, 7$; $j = 1, 2, 3, 4$). Table 3 represents the above given the data assessed by operations managers. Therefore, the following section of this paper proposes a two-stage assessment for evaluating the degree of MF.

4. Two-Stage Assessment

Here, a two-stage assessment for evaluating the degree of MF is considered. This assessment aggregates the first-stage aggregate assessment on dimensions of MF and then aggregates them as the degree of MF.

4.1 First-stage assessment

Step 1: Assessing the importance grade and related performance rating. The performance rating is rated against flexibility metrics, and importance of flexibility dimensions are also linguistic terms, presented in Table 1, established by operations managers.

Step 2: Making the information uniform. The importance grade and related performance rating must be transformed into a unified linguistic term set V, that a fuzzy assessment matrix for $X_i \times V$ can be formed. For instance, let $X_i = X_2$. Then a fuzzy assessment matrix $M(X_2)$ is obtained as follows:

$$M(X_2) = \begin{array}{c} \\ X_{21} \\ X_{22} \\ X_{23} \\ X_{24} \end{array} \begin{array}{c} \begin{array}{cccc} v_0 & v_1 & \cdots & v_{10} \end{array} \\ \begin{pmatrix} u(P_{21}, v_0) & u(P_{21}, v_1) & \cdots & u(P_{21}, v_{10}) \\ u(P_{22}, v_0) & u(P_{22}, v_1) & \cdots & u(P_{22}, v_{10}) \\ u(P_{23}, v_0) & u(P_{23}, v_1) & \cdots & u(P_{23}, v_{10}) \\ u(P_{24}, v_0) & u(P_{24}, v_1) & \cdots & u(P_{24}, v_{10}) \end{pmatrix} \end{array} . \quad (3)$$

Table 4. The contents of structure model

Flexibility dimension (with 'Q_2')	X_1	X_2	X_3	X_4	X_5	X_6	X_7
Importance grade (I)	I_1	I_2	I_3	I_4	I_5	I_6	I_7
Flexibility metric (with 'Q_1')	$X_{11}X_{12}X_{13}X_{14}$	$X_{21}X_{22}X_{23}X_{24}$	$X_{31}X_{32}X_{33}X_{34}$	$X_{41}X_{42}X_{43}X_{44}$	$X_{51}X_{52}X_{53}X_{54}$	$X_{61}X_{62}X_{63}X_{64}$	$X_{71}X_{72}X_{73}X$
Performance rating (P)	$P_{11}\,P_{12}\,P_{13}\,P_{14}$	$P_{21}\,P_{22}\,P_{23}\,P_{24}$	$P_{31}\,P_{32}\,P_{33}\,P_{34}$	$P_{41}\,P_{42}\,P_{43}\,P_{44}$	$P_{51}\,P_{52}\,P_{53}\,P_{54}$	$P_{61}\,P_{62}\,P_{63}\,P_{64}$	$P_{72}\,P_{71}\,P_{73}\,P$

By the same way, we can form fuzzy assessment matrices $M(X_1)$, $M(X_3)$, $M(X_4)$, $M(X_5)$, $M(X_6)$ and $M(X_7)$ for X_1, X_3, X_4, X_5, X_6, X_7, respectively. Similarly, importance grades of flexibility dimensions are also transformed into fuzzy assessment vectors for $I_i \times V$, as follows:

$$I1(I_i) = (u(I_i, v_0), u(I_i, v_1), \cdots, u(I_i, v_{10}),) \text{ for } i = 1, 2, \cdots, 7. \quad (4)$$

Step 3: Evaluating the first-stage aggregative assessment for flexibility dimension. Using the concept of fuzzy majority over the flexibility metrics specified by a linguistic quantifier Q_1, and applying the maximum entropy ordered weighted averaging (MEOWA) operator, defined by Yager (1988) and O'Hagan (1988), yields the first-stage aggregative assessment on dimension of flexibility, as follows: Let

$$F1(X_i, v_k) = \Phi_{Q1}\,(u(P_{i1},v_k), u(P_{i2},v_k), u(P_{i3},v_k), u(P_{i4},v_k)) \quad (5)$$

$$\text{for } i = 1, 2, \cdots, 7,\ k = 0, 1, \cdots, 10.$$

Then, the vector of the first-stage aggregative assessment for dimension X_i, $F1(X_i)$ is defined as

$$F1(X_i) = (F1(X_i, v_0), F1(X_i, v_1), \cdots, F1(X_i, v_{10})) \text{ for } i = 1, 2, \cdots, 7. \quad (6)$$

Step 4: Defuzzified by the centroid method. The first-stage aggregative rating and importance for flexibility dimension, $D(X_i)$ and $I(X_i)$ are defuzzified by the centroid method:

$$D(X_i) = \sum_{k=0}^{10} VG(k) \times F1(X_i, v_k) / \sum_{k=0}^{10} F1(X_i, v_k) \text{ for } i = 1, 2, \cdots, 7, \quad (7)$$

$$I(X_i) = \sum_{k=0}^{10} VG(k) \times u(I_i, v_k) / \sum_{k=0}^{10} u(I_i, v_k) \text{ for } i = 1, 2, \cdots, 7. \quad (8)$$

Step 5: Calculating the differences for the flexibility dimension. Computing the differences between the $I(X_i)$ and $D(X_i)$ with respect to each flexibility dimension, $\gamma(X_i)$ is defined as

$$\gamma(X_i) = I(X_i) - D(X_i) \text{ for } i = 1, 2, \cdots, 7. \quad (9)$$

Comparing the differences of each dimension may yield maximum positive values and then determine which dimensions of MF represent the best directions for improvement.

4.2 Second-stage assessment

Step 6: Evaluating the degree of MF. As mentioned above, both the importance grade and the first-stage aggregative assessment on each flexibility dimension should be evaluated to determine the degree of MF. Using the concept of fuzzy majority over the flexibility dimensions specified by a linguistic quantifier Q_2, and applying the MEOWA operator to yields the degree of MF, as follows: Let

$$I2(MF, v_k) = \Phi_{Q2}(u(I_1, v_k), u(I_2, v_k), \cdots, u(I_7, v_k)) \text{ for } k = 0, 1, \cdots, 10, \quad (10)$$

$$F2(MF, v_k) = \Phi_{Q2}(F1(X_1, v_k), F1(X_2, v_k),, \cdots, F1(X_7, v_k)) \text{ for } k = 0, 1, \cdots, 10. \quad (11)$$

Then, the vectors of the second-stage aggregative assessment for MF, F2(MF) and I2(MF) are defined as:

$$I2(MF) = (I2(MF, v_0), I2(MF, v_1), \cdots, I2(MF, v_{10})), \quad (12)$$

$$F2(MF) = (F2(MF, v_0), F2(MF, v_1), \cdots, F2(MF, v_{10})), \quad (13)$$

where the I2(MF) and F2(MF) represent the importance of MF and the degree of MF, respectively, according to the assessments of operations mangers.

Step 7: Defuzzified by the centroid method. The second-stage aggregative rating and importance for flexibility dimension, D(MF) and I(MF) are defuzzified by the centroid method:

$$D(MF) = \sum_{k=0}^{10} VG(k) \times F2(MF, v_k) / \sum_{k=0}^{10} F2(MF, v_k), \quad (14)$$

$$I(MF) = \sum_{k=0}^{10} VG(k) \times I2(MF, v_k) / \sum_{k=0}^{10} I2(MF, v_k). \quad (15)$$

Step 8: Calculating the difference for MF. Thus, the difference between I(MF) and D(MF), γ (MF), is also computed. The decision maker may decide whether there is a need to improve MF or not.

4.3 Numerical example

The following the two-stage assessment is applied to measure the degree of MF. Suppose that managers have identified the importance grade and related performance rating, as presented in Table 4.

By the first-stage assessment: Making the information uniform, we obtain

$$M(X_2) = \begin{array}{c} \\ X_{21} \\ X_{22} \\ X_{23} \\ X_{24} \end{array} \begin{array}{c} \begin{array}{ccccccccccc} V_0 & V_1 & V_2 & V_3 & V_4 & V_5 & V_6 & V_7 & V_8 & V_9 & V_{10} \end{array} \\ \begin{pmatrix} 0 & 0 & 0.5 & 1 & 1 & 0.5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 1 & 1 & 0.5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & 1 & 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 1 & 1 & 0.5 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}.$$

Similarly, we have

$$M(X_1)=\begin{pmatrix} 0\,0\,0\,0\,0.5\,1\,0.5\,0\,0\,0\,0 \\ 0\,0\,0\,0\,0\,0.5\,1\,1\,0.5\,0\,0 \\ 0\,0\,0\,0\,0\,0.5\,1\,1\,0.5\,0\,0 \\ 0\,0\,0\,0\,0.5\,1\,0.5\,0\,0\,0\,0 \end{pmatrix}, \quad M(X_3)=\begin{pmatrix} 0.5\,1\,1\,0.5\,0\,0\,0\,0\,0\,0\,0 \\ 0\,0\,0.5\,1\,1\,0.5\,0\,0\,0\,0\,0 \\ 0\,0\,0\,0\,0.5\,1\,0.5\,0\,0\,0\,0 \\ 0\,0\,0.5\,1\,1\,0.5\,0\,0\,0\,0\,0 \end{pmatrix},$$

$$M(X_4)=\begin{pmatrix} 0\,0\,0\,0\,0\,0.5\,1\,1\,0.5\,0\,0 \\ 0\,0\,0\,0\,0\,0\,0\,0.5\,1\,1\,0.5 \\ 0\,0\,0\,0\,0\,0.5\,1\,1\,0.5\,0\,0 \\ 0\,0\,0\,0\,0\,0.5\,1\,1\,0.5\,0\,0 \end{pmatrix}, \quad M(X_5)=\begin{pmatrix} 1\,0.5\,0\,0\,0\,0\,0\,0\,0\,0\,0 \\ 0\,0\,0.5\,1\,1\,0.5\,0\,0\,0\,0\,0 \\ 0.5\,1\,1\,0.5\,0\,0\,0\,0\,0\,0\,0 \\ 0.5\,1\,1\,0.5\,0\,0\,0\,0\,0\,0\,0 \end{pmatrix},$$

$$M(X_6)=\begin{pmatrix} 0\,0\,0\,0\,0\,0\,0\,0.5\,1\,1\,0.5 \\ 0\,0\,0\,0\,0\,0\,0\,0\,0\,0.5\,1 \\ 0\,0\,0\,0\,0\,0.5\,1\,1\,0.5\,0\,0 \\ 0\,0\,0\,0\,0\,0\,0\,0.5\,1\,1\,0.5 \end{pmatrix}, \quad M(X_7)=\begin{pmatrix} 0\,0\,0\,0\,0.5\,1\,0.5\,0\,0\,0\,0 \\ 0\,0\,0\,0\,0\,0.5\,1\,1\,0.5\,0\,0 \\ 0\,0\,0.5\,1\,1\,0.5\,0\,0\,0\,0\,0 \\ 0\,0\,0\,0\,0.5\,1\,0.5\,0\,0\,0\,0 \end{pmatrix}, \text{ and}$$

$$I1(I) = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0.5 & 1 & 1 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 1 & 1 & 0.5 \\ 0 & 0 & 0 & 0 & 0 & 0.5 & 1 & 1 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & 1 & 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & 1 & 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.5 & 1 & 1 & 0.5 & 0 & 0 \\ 0 & 0 & 0.5 & 1 & 1 & 0.5 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Using the linguistic quantifier ‘as many as possible’ with the pair (0.5, 1). The algorithm for calculating the maximum entropy weights of MEOWA operator (see the Appendix) yields the weights W_1, the orness α_1 and the maximum entropy weights W_1^*, as follows,

$W_1 = [0, 0, \mathbf{0.5}, 0.5]$, in which

$W_1(3) = Q(3/4) - Q(2/4) = ((0.75-0.5)/(1-0.5)) - ((0.5-0.5)/(1-0.5)) = 0.5$,

$\alpha_1 = ((4-1)\times 0 + (4-2)\times 0 + (4-3)\times 0.5 + (4-4)\times 0.5) / (4-1) = 0.1667$,

$W_1^* = [0.0311, 0.0856, 0.2355, 0.6478]$.

Table 4. Assessing data

Flexibility dimension (‘as many as possible’)	X_1	X_2	X_3	X_4	X_5	X_6	X_7
Importance grade (I)	H	VH	H	M	M	H	L
Flexibility metric (‘as many as possible’)	X_{11} X_{12} X_{13} X_{14}	X_{21} X_{22} X_{23} X_{24}	X_{31} X_{32} X_{33} X_{34}	X_{41} X_{42} X_{43} X_{44}	X_{51} X_{52} X_{53} X_{54}	X_{61} X_{62} X_{63} X_{64}	X_{71} X_{72} X_{73} X_{74}
Performance rating(P)	M H H M	L L M L	VL L M L	H VH H H	DL L VL VL	VH DH H VH	M H L M

Then the first-stage aggregative assessments for each flexibility dimension obtained are:

$F1(X_1)$ = (0, 0, 0, 0, 0.0584, 0.5584, 0.5584, 0.2344, 0.0584, 0, 0),

$F1(X_2)$ = (0, 0, 0.1761, 0.3522, 0.6761, **0.5156**, 0.0156, 0, 0, 0, 0),

$F1(X_3)$ = (0.0156, 0.0311, 0.1916, 0.2344, 0.2344, 0.1916, 0.0156, 0, 0, 0, 0),

$F1(X_4)$ = (0, 0, 0, 0, 0, 0.1761, 0.3522, 0.6761, 0.5156, 0.0311, 0.0156),

$F1(X_5)$ = (0.1916, 0.2344, 0.2344, 0.1916, 0.0311, 0.0156, 0, 0, 0, 0, 0),

$F1(X_6)$ = (0, 0, 0, 0, 0, 0.0156, 0.0311, 0.1916, 0.2344, 0.2344, 0.1916),

$F1(X_7)$ = (0, 0, 0, 0, 0.0156, 0.5156, 0.6761, 0.3522, 0.1761, 0, 0),

where, for example, the value $F1(X_2, v_5)$ is obtained according to this expression:

$F1(X_2, v_5) = \Phi_{Q1}(0.5, 0.5, 1, 0.5) = 0.5156$.

Defuzzified by the centroid method, we have

$D(X_1)$=0.5779, $D(X_2)$ = 0.3909, $D(X_3)$=0.3404, $D(X_4)$=0.6952, $D(X_5)$=0.1718,

$D(X_6)$=0.8282, $D(X_7)$=0.5.

By the same way, we have

$I(X_1)$=0.65, $I(X_2)$=0.8444, $I(X_3)$=0.65, $I(X_4)$=0.5, $I(X_5)$=0.5, $I(X_6)$=0.65, $I(X_7)$=0.35.

The differences on each dimension are shown in Table 5, and then determines dimension X_2 as the best direction for improvement.

By the second-stage assessment: Using the linguistic quantifier ‘as many as possible’ with the pair (0.5, 1), yields the weights W_2, the orness α_2 and the maximum entropy weights W_2^*, as follows,

Table 5. The result of first-stage assessment

Flexibility dimension	X_1	X_2	X_3	X_4	X_5	X_6	X_7
Importance grade (I)	0.65	0.8444	0.65	0.5	0.5	0.65	0.35
(2) First-stage aggregative rating	0.5799	0.3903	0.3404	0.6975	0.1718	0.8282	0.5
Difference (1) — (2)	+0.0701	+0.4541	+0.3096	-0.1975	-0.3282	-0.1782	-0.15

W_2 = [0, 0, 0, 0.1429, 0.2857, 0.2857, 0.2857], α_2=0.2143,

W_2^* = [0.0187, 0.0313, 0.0525, 0.0880, 0.1476, 0.2473, 0.4145].

Then the vectors of the second-stage aggregative assessment for MF obtained are:

I2(MF) = (0, 0, 0.0094, 0.0187, 0.0606, 0.3178, 0.2204, 0.1466, 0.1047, 0.0187, 0.0094),

F2(MF)=(0.0041,0.0054,0.0210,0.0267,0.0398,0.1082,0.0404,0.0328,0.0214,0.0054, 0.0041),

where, for example, the value F2(MF, v_5) is obtained according to this expression:

F2(MF, v_5)= Φ_{Q2}(0.5584, 0.5156, 0.1916, 0.1761, 0.0156, 0.0156, 0.5584) = 0.1082.

Defuzzified by the centroid method, we have

D(MF) = 0.5045 and I(MF) = 0.5905.

Therefore, 0.5905 and 0.5045 are the linguistic assessments for the importance of MF and the degree of MF, respectively. The difference for MF is obtained, as follows:

γ (MF) = +0.086.

Therefore, the decision maker may identify a need to improve MF, and determine dimension X_2 as the best direction for improvement.

5. Reasonability of the Two-Stage Assessment

According to the monotonic property, in this paper some examples are thus represented to shown that this method is reasonable.Since the importance grade and related performance rating are corresponding in ranks, we divide them into four classes, then Table 6 can be obtained.

Table 6. The combination of the performance rating and the importance grade

	Performance rating (P) / Importance grade (I)			
	Low	→		High
	DL , VL	L, M	H	VH, DH
Degree of manufacturing flexibility (D(MF))	(b)	(c)	(d)	(e)
Importance of manufacturing flexibility (I(MF))	(b')	(c')	(d')	(e')

Table 7. The combination of importance grade (I), performance rating (P), degree of manufacturing flexibility(D(MF)) and importance of manufacturing flexibility(I(MF))

Dimension	X_1				X_2				X_3				X_4				X_5				X_6				X_7				
Metric	X_{11}	X_{12}	X_{13}	X_{14}	X_{21}	X_{22}	X_{23}	X_{24}	X_{31}	X_{32}	X_{33}	X_{34}	X_{41}	X_{42}	X_{43}	X_{44}	X_{51}	X_{52}	X_{53}	X_{54}	X_{61}	X_{62}	X_{63}	X_{64}	X_{71}	X_{72}	X_{73}	X_{74}	D(MF)
(a) P(1)	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0.0555
(b) P(1, 2)	1	2	1	2	1	2	1	2	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	1	1	2	2	0.0840
(c) P(3, 4)	3	4	3	4	3	4	3	4	3	3	3	3	3	3	3	3	4	4	4	4	4	4	4	4	3	3	4	4	0.4177
(d) P(5)	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	0.6500
(e) P(6, 7)	7	6	7	6	7	6	7	6	7	7	7	7	7	7	7	7	6	6	6	6	6	6	6	6	7	7	6	6	0.8933
(f) P(7)	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	0.9445

Dimension	X_1	X_2	X_3	X_4	X_5	X_6	X_7	
Metric	$X_{11}X_{12}X_{13}X_{14}$	$X_{21}X_{22}X_{23}X_{24}$	$X_{31}X_{32}X_{33}X_{34}$	$X_{41}X_{42}X_{43}X_{44}$	$X_{51}X_{52}X_{53}X_{54}$	$X_{61}X_{62}X_{63}X_{64}$	$X_{71}X_{72}X_{73}X_{74}$	I(MF)
(a') I(1)	1	1	1	1	1	1	1	0.0555
(b') I(1, 2)	1	2	1	2	1	2	1	0.0852
(c') I(3, 4)	4	3	4	3	4	3	4	0.4184
(d') I(5)	5	5	5	5	5	5	5	0.6500
(e') I(6, 7)	7	6	7	6	7	6	7	0.9148
(f') I(7)	7	7	7	7	7	7	7	0.9445

In order to show the monotonic property, suppose that decision makers assign a fixed linguistic fuzzy quantifier to the corresponding flexibility dimensions and metrics. Then the combination of

Table 8. Degree of manufacturing flexibility in two ways table
(performance rating and linguistic quantifier)

		Performance rating (P)			
	Linguistic quantifier	(DL, VL) Low	(L, M) Middle low	(H) Middle high	(VH, DH) High
D(MF)	As many as possible (0.5, 1)	0.0840	0.4177	0.65	0.8933
I(MF)	As many as possible (0.5, 1)	0.0852	0.4184	0.65	0.9148

the performance rating, the importance grade, and related degree of MF are presented in Table 7, from (a) to (f). Moreover, Table 8 represented the degree of MF of each entry of Table 8, and show that all the entries on the diagonals from the left to the right are increasing. It conceded with what we expected.

6. Conclusion

This paper has presented a fuzzy fusion method of linguistic information applied in a multiple attributes decision-making model to improve the manufacturing flexibility evaluation process. Based on the concept of linguistic quantifier chosen by decision makers, the model constructs fuzzy linguistic assessments among manufacturing systems and determines ranking orders among them. While evaluating the degree of manufacturing flexibility, one may find the need for improving manufacturing flexibility, and determine the dimensions of manufacturing flexibility as the best directions to improvement until decision makers can accept it. The importance grades or performance ratings must be improved until acceptable when evaluating the degree of manufacturing flexibility. We also show that the linguistic assessment of manufacturing flexibility is reasonable.

Appendix: Algorithm for calculating the MEOWA weights (Filev and Yager, 1995; Yager, 1988)

Step1: Determine the non-decreasing proportional linguistic quantifier Q, used to represent the fuzzy majority over dimensions or metrics, as follows,

$$Q(r) = \begin{cases} 0 & \text{if } r < a, \\ (r-a)/(b-a) & \text{if } a \leqq r \leqq b, \\ 1 & \text{if } r > b, \end{cases}$$

with a, b, r $\in$ [0, 1]. Some non-decreasing proportional linguistic fuzzy quantifiers are

typified by terms ‘most’, ‘at least half’, ‘as many as possible’, for example. Respective parameters (a, b) are (0.3, 0.8), (0, 0.5) and (0.5, 1), respectively.

Step 2: Compute the weights W as follows,

$w_i = Q(i/n) - Q((i-1)/n), \qquad i = 1, 2, \cdots, n.$

Step 3: Compute the orness measure α as follows.

$\alpha = (\sum_{i=1}^{n} (n-i) w_i) / (n-1).$

Step 4: Compute the maximum entropy weights W^*, which are used in modified LOWA operator, according to the two-step process.

4-1: Find a positive solution h^* of the algebraic equation,

$$\sum_{i=1}^{n} ((n-i)/(n-1) - \alpha) h^{(n-i)} = 0.$$

4-2: Obtain W^* from the following equation, using $\beta^* = (n-1)\ln h^*$,

$$w_i^* = \frac{e^{\beta^* \times ((n-i)/(n-1))}}{\sum_{i=1}^{n} e^{\beta^* \times ((n-j)/(n-1))}}, \qquad i = 1, 2, \cdots, n, \quad j = 1, 2, \cdots, n.$$

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