A fuzzy multi-attribute group decision-making approach for evaluating the flexibility in an advanced manufacturing system

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Abstract

This paper presents a fuzzy multi-attribute decision-making algorithm for evaluating flexibility in an advanced manufacturing system development. This evaluation problem is formulated as a multi-attribute decision-making model in a fuzzy environment and solved by a fusion method based on the MEOWA operators. While evaluating the degree of manufacturing flexibility, one may find the need for improving manufacturing flexibility, and determine the dimensions of manufacturing flexibility as the best directions to the improvement of manufacturing flexibility until she/he can accept it. We also show that higher the combination of importance grade and performance rating the higher the degree of manufacturing flexibility.

模糊多屬性群體決策方法應用於先進製造系統彈性評估

摘要

針對先進製造系統的彈性評估，本論文提出一種模糊多屬性決策程序，來支援作業管理者進行製造彈性管理。首先，將製造彈性評估問題建構為一個模糊環境下多屬性決策模式，並提出一種運用 MEOWA 運算子的語意融合演算法於評估製造彈性。在評估製造彈性時，不僅可以發現改善製造彈性的需要，更可以確定製造彈性的改善方向，直到評估者滿意為止。最後，例釋與模擬本方法，顯示出本評估結果是合理的。

1. Introduction

Manufacturing environments have changed so fast in recent decades that the flexibility of advanced manufacturing systems has become increasingly important. Flexible manufacturing systems, computer-integrate manufacturing systems, Just-in-Time systems, flexible factories, and so forth, are rely on manufacturing flexibility. Generally, manufacturing flexibility (MF) is the ability of a manufacturing system to cope with environmental changes effectively and efficiently. MF has been emphasized as a major competitive priority in manufacturing system. Flexibility improvement is an important issue on the operations managers that must be evaluating the degree of MF when making capital investment decisions and measuring performance[5]. However, MF is a complex, multidimensional and difficult-to-synthesize concept[14], and the needs of operations managers have not yet been met.

Many researchers have considered definitions, requests, classificatory in dimensions, measurement, choices, and interpretations of MF[1,3,7,13,14]. Upton[16] proposed a framework for analyzing MF according to different dimensions, each of which to cope with the environmental changes at different time intervals and is specified by three elements: range, mobility and uniformity. Golden and Powell[6] presented an inclusive definition in which flexibility can be measured by four metrics: efficiency, responsiveness, versatility and robustness. Many researchers have tried objectively to evaluate MF. Several efforts are theoretical and involve only two or three dimensions[1]. The importance grade for flexibility dimensions, and the subjective evaluation of MF have seldom been addressed.

Most operations managers cannot give exact numerical values to represent opinions, based on human perception, on flexibility metrics, more realistic evaluation uses linguistic assessments rather than numerical values[1,5, 9, 17]. In fact the flexibility metrics are specified as linguistic terms, such as very high, high, middle, low, and very low. After Zadeh[20] introduced fuzzy set theory to deal with vague problems, linguistic terms have been used for approximate reasoning within the framework of fuzzy set theory to handle the ambiguity of evaluating data and the vagueness of linguistic expression[21]. Normal trapezoidal fuzzy numbers have been used to characterize linguistic terms used in approximate reasoning. Wang and Chuu[18] propose a decision-making model for determining the degree of MF using a fuzzy linguistic approach, which based on the direct computation on linguistic labels.

In the fuzzy linguistic approach, the cardinality of the linguistic label set is an important factor to determine the uncertainty of the evaluation information[19]. According to the uncertainty of the evaluator's information, the linguistic label set chosen will have more or less labels. Herrera at al.[9] presented a fusion approach of multi-granularity linguistic information for managing different linguistic label sets, which are applied to decision-making problems with numerous information sources that may be experts or criteria.

Therefore, based on the algorithm developed in Wang and Chuu[18] and the fusion of linguistic information in Herrera at al.[9], the purpose of this paper was to constructs a decision-making structure model for evaluating the flexibility of a manufacturing system. An algorithm is proposed to determine the degree of MF in a fuzzy environment using a fusion method of evaluation information to any phase of a manufacturing system development. Section 2 presents a fusion method of linguistic information. Section 3 presents a hierarchical structure model of MF. Section 4 describes a two-stage MF evaluation algorithm. Finally, the sensitivity analysis show that higher the combination of importance grade and performance rating the higher the degree of MF.

2. Fusion of linguistic information

The fuzzy linguistic approach assesses the linguistic variables using words or sentences in natural language[21]. This approach is appropriate for some problems in which information may be qualitative, or quantitative information may not be stated precisely, since either it is unavailable or the cost of its computation is prohibitive, such that an 'approximate value' suffices[9]. In applying a fuzzy linguistic approach to the measurement of manufacturing flexibility, only the performance of flexibility metrics are classified into low, middle or high and neglect the important of the flexibility that will induce the imprecision and bias. Therefore, importance for each flexibility metric should be evaluating to get the degree of MF in a manufacturing system.

2.1 Combination of linguistic labels

In the fuzzy linguistic approach, the performance rating and importance grade should be evaluated for each flexibility metrics. Consequently, both were scored on a linguistic scale. The strongest assessment for various metric is given the highest (lowest) linguistic label 'definitely high' ('definitely low') in the linguistic scale, which consisting of various linguistic labels. For example, let S={s_0, s_1,…, s_6} be a finite and totally ordered label set on [0, 1] , as shown in Table 1, where the middle label s_3 represents 'average', and the remaining labels are ordered symmetrically around s_3, and exhibit the following properties[10].

1. The set is ordered: $s_i \geqq s_j$ if $i \geqq j$.
2. The negation operator is defined as Neg (s_i) = s_j such that j = 6-i.

Table 1. Linguistic variables of performance rating and importance grade

Seven ranks of performance rating	Fuzzy number	Seven ranks of importance grade	Fuzzy number
1: s_0 = Definitely low（DL）	（0, 0, 0, 0.1）	1: s_0 = Definitely low（DL）	（0, 0, 0, 0.1）
2: s_1 = Very low（VL）	（0, 0.1, 0.2, 0.3）	2: s_1 = Very low（VL）	（0, 0.1, 0.2, 0.3）
3: s_2 = Low（L）	（0.2,0.3,0.4,0.5）	3: s_2 = Low（L）	（0.2,0.3,0.4,0.5）
4: s_3 = Middle（M）	（0.4,0.5,0.5,0.6）	4: s_3 = Middle（M）	（0.4,0.5,0.5,0.6）
5: s_4 = High（H）	（0.5,0.6,0.7,0.8）	5: s_4 = High（H）	（0.5,0.6,0.7,0.8）
6: s_5 = Very high（VH）	（0.7,0.8,0.9,1.0）	6: s_5 = Very high（VH）	（0.7,0.8,0.9,1.0）
7: s_6 = Definitely high(DH）	（0.9,1.0,1.0,1.0）	7: s_6 = Definitely high（DH）	（0.9,1.0,1.0,1.0）

3. The maximization operator is Max (si, sj) = si if si $\geqq$ sj.
4. The minimization operator is Min (s_i, s_j) = s_i if $s_i \leqq s_j$.

This work applies a convex combination of linguistic labels by direct computation on labels; that is, the independently of the semantics of the label set. The convex combination of labels is defined by Delgado et al.[2]. Its property is presented in [2] and its use in linguistic ordered weighted averaging (LOWA) operator in [8, 9, 10].

Let A＝{p_1, p_2,…, p_m} be a set of linguistic labels to be aggregated, and each element $p_i \in A$ is the i^{th} largest label. The convex combination of these m labels is given by

$$C^m\{w_k, p_k, k = 1, 2,\ldots, m\} = w_1 \odot p_1 \oplus (1-w_1) \odot C^{m-1}\{\beta_h, p_h, h = 2, 3,\ldots, m\},$$

where W = [$w_1, w_2,\ldots,w_m$], is a weighting vector, such that, $w_i \in [0, 1]$ and $\Sigma_i w_i = 1$, $\beta_h = w_h / \Sigma_h w_h$, h = 2, 3,…, m, $\odot$ is the general product of a label by a positive real number and $\oplus$ is the general addition of labels defined in [2]. If m = 2, then C^2 is defined as

$$C^2\{w_i, p_i, i = 1, 2\} = w_1 \odot s_j \oplus (1- w_1) \odot s_i = s_k, s_j, s_i \in S\ (j \geqq i)$$

$$\text{such that } k = Min\ (6, i + round\ (w_1 \times (j - i))),$$

where 'round' is the usual round operation, and $p_1 = s_j$, $p_2 = s_i$.

Table 2. Linguistic values and fuzzy numbers of criteria rating of flexibility

Eleven ranks of criteria rating of flexibility	Fuzzy number
v_0：Definitely low	(0.0, 0.0, 0.0, 0.1)
v_1：Extra low	(0.0, 0.1, 0.1, 0.2)
v_2：Very low	(0.1, 0.2, 0.2, 0.3)
v_3：Low	(0.2, 0.3, 0.3, 0.4)
v_4：Slightly low	(0.3, 0.4, 0.4, 0.5)
v_5：Middle	(0.4, 0.5, 0.5, 0.6)
v_6：Slightly high	(0.5, 0.6, 0.6, 0.7)
v_7：High	(0.6, 0.7, 0.7, 0.8)
v_8：Very high	(0.7, 0.8, 0.8, 0.9)
v_9：Extra high	(0.8, 0.9, 0.9, 1.0)
v_{10}：Definitely high	(0.9, 1.0, 1.0, 1.0)

2.2 Making the information uniform

The criteria ratings of flexibility are linguistic variables with 11 labels v_0, v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, which are treated as fuzzy number with trapezoidal membership function $\mu(x)$, which associates with each element x in X a real number in the interval [0, 1], as shown in Table 2. The defuzzification by the centroid method is defined as

$$\int_a^b x\mu(x)dx \Big/ \int_a^b \mu(x)dx,$$

where a and b are lower and upper limits of the integral, respectively. This work have its centroid VG(0)=0.0333, VG(1)=0.1, VG(2)=0.2, VG(3)=0.3, VG(4)=0.4, VG(5)=0.5, VG(6)=0.6, VG(7)=0.7, VG(8)=0.8, VG(9)=0.9, VG(10)=0.9667 as center of mass of v_0, v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, respectively.

Let V={v_0, v_1, …, v_{10}} and S={s_0, s_1, …, s_6} be two linguistic label sets, with 11 and 7 labels, respectively, as shown in Figure 1. According to Herrera et al.[9], the former is a specific linguistic domain, which is a basic linguistic label set with the maximum cardinality, chosen so as not to impose useless precision to the original evaluations. The latter is a linguistic label set to express the initial evaluation values, which is assessed in different linguistic label sets, assigned to the criteria. The evaluation values expressed using various cardinality of S should be converted into V in order to allow an appropriate discrimination of the original evaluation values. By the transformation function defined by Herrera et al. [9], a fuzzy assessment matrix for S×V can be formed. The transformation function θ_{SV} is defined as

$$\theta_{SV}: S \rightarrow F(V),$$

$$\theta_{SV}(s_i)=\{(u_{ij}, v_j)/j \in \{0, 1, \ldots, 10\}\} \text{ for } s_i \in S,$$

$$u_{ij} = \max_x \min\{\mu_{si}(x), \mu_{vj}(x)\},$$

where F(V) is the set of fuzzy sets defined in V, and $\mu_{si}(x)$ and $\mu_{vj}(x)$ are the membership functions of the fuzzy sets associated to the labels s_i and v_j, respectively.

2.3 The maximum entropy ordered weighted averaging operators

Yager[19] introduced ordered weighted averaging (OWA) operators to provide a family of aggregation operators that always lies between the 'and' and the 'or'. These operators are characterized by the OWA weighting vector, which were associated with two measures: 'orness' and 'entropy'. The orness measure characterizes the degree to which the aggregation is a Max-like or a Min-like operation, and the entropy measure uses the Shannon information concept, the more entropy the more of the information about the individual aggregates is being used in the aggregation process. In the OWA operators, the concept of fuzzy majority can be incorporated by means of a non-decreasing proportional linguistic quantifier, such as 'most', 'at least half', 'as many as possible', used to compute the weighting vector.

O'Hagan[12] developed a method to obtaining the maximum entropy OWA (MEOWA) weighting vector that have a predefined degree of orness and that maximize the entropy. This approach is based upon the solution of a constraint optimization problem. Filev and Yager[4] suggested a two-step process used for obtain the MEOWA weighting vector that generate some prescribed degree of orness without having to solve the constraint optimization problem. Mitchell and Estrakh[11] presented an application of the MEOWA operators to lossless image compression.

The algorithm for calculating a MEOWA weighting vector as follows

Step1: Determine the non-decreasing proportional linguistic quantifier Q, used to represent the fuzzy majority over dimensions or metrics,

$$Q(r) = \begin{cases} 0 & \text{if } r < a, \\ (r-a)/(b-a) & \text{if } a \leqq r \leqq b, \\ 1 & \text{if } r > b, \end{cases}$$

with a, b, r $\in$ [0, 1]. For example, some non-decreasing proportional linguistic quantifiers are typified by terms 'most', 'at least half', and 'as many as possible', respective parameters (a, b) are (0.3, 0.8), (0, 0.5) and (0.5, 1), respectively.

Step 2: Compute the weighting vector W,

$$W(i) = Q(i/n) - Q((i-1)/n), \text{ for } i = 1, 2, \ldots, n.$$

Step 3: Compute the orness α ,

$$\alpha = \left(\sum_{i=1}^{n} (n-i)\, W(i)\right) / (n-1).$$

Step 4: Compute the maximum entropy weighting vector W*, which is used in MEOWA operators, according to the two-step process.

4-1: Find a positive solution h^* of the algebraic equation,

$$\sum_{i=1}^{n} ((n-i)/(n-1) - \alpha)\, h^{(n-i)} = 0.$$

4-2: Obtain W^* from the following equation, using $\beta^* = (n-1)\ln h^*$,

$$W^*(i) = \frac{e^{\beta^* \times ((n-i)/(n-1))}}{\sum_{j=1}^{n} e^{\beta^* \times ((n-j)/(n-1))}} \qquad \text{for } i = 1,2,\ldots,n$$

3. Hierarchical Structure model of manufacturing flexibility

A systematic approach is proposed to evaluate the degree of MF, using a fuzzy set theory and hierarchical structure analysis. This method is suitable for decision-making in a fuzzy environment. The dimensions of flexibility proposed by Gerwin[6], Slack[15] were expressed seven dimensions as mix flexibility, changeover flexibility, modification flexibility, volume flexibility, rerouting flexibility, material flexibility and sequencing flexibility, based on the relationship between MF and environmental changes. Furthermore, each dimension was divided Figure 1; for example, efficiency was denoted by X_{i1}, responsiveness by X_{i2}, and so on.

The decision-makes consider the importance grade and related performance rating, grading both as S={s_0, s_1,…, s_6}. Suppose the degree of MF is responsible for assessing by operations managers, such as vice president of manufacturing, plant manager or management consultant, and so on, whose collective experience extended across a broad range of manufacturing environment and its environmental changes. The symbol I_i is used to denote the grade of importance of dimension X_i; P_{ij} and I_{ij} be the performance rating and importance grade of flexibility metric X_{ij}, respectively, according to operations manager's assessing data (i=1, 2,…, 7; j=1, 2, 3, 4). Table 3 represents the

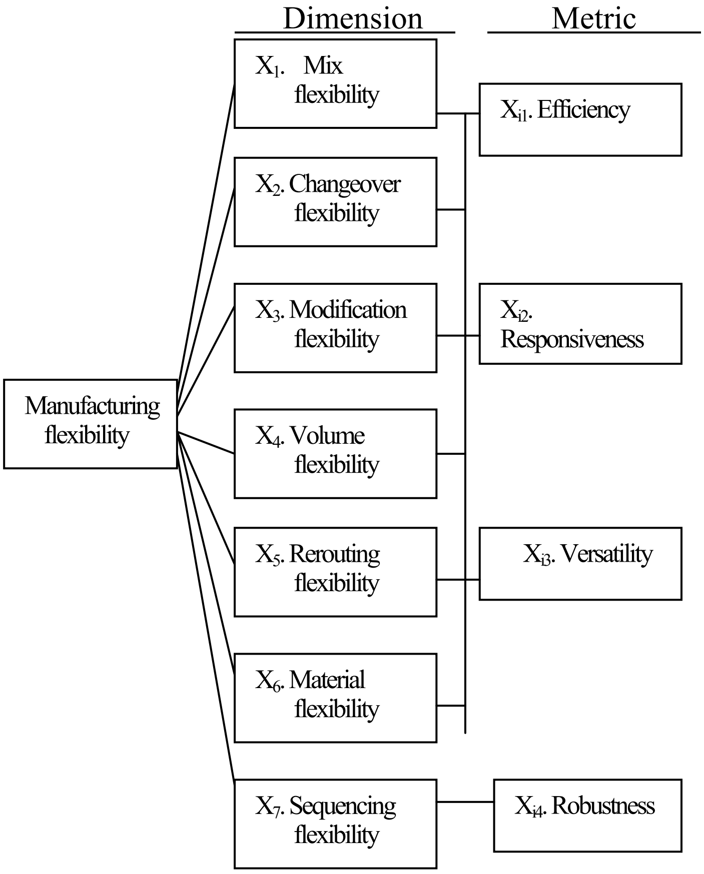

Figure 1. Hierarchical structure model of manufacturing flexibility

Table 3. The contents of structure model

Flexibility dimension (with Q_2')	X_1	X_2	X_3	X_4	X_5	X_6	X_7
Importance grade (GI)	I_1	I_2	I_3	I_4	I_5	I_6	I_7
Flexibility metric (with 'Q_1')	$X_{11}X_{12}X_{13}X_{14}$	$X_{21}X_{22}X_{23}X_{24}$	$X_{31}X_{32}X_{33}X_{34}$	$X_{41}X_{42}X_{43}X_{44}$	$X_{51}X_{52}X_{53}X_{54}$	$X_{61}X_{62}X_{63}X_{64}$	$X_{71}X_{72}X_{73}X_{74}$
Performance rating (P)	P_{11} P_{12} P_{13} P_{14}	P_{21} P_{22} P_{23} P_{24}	P_{31} P_{32} P_{33} P_{34}	P_{41} P_{42} P_{43} P_{44}	P_{51} P_{52} P_{53} P_{54}	P_{61} P_{62} P_{63} P_{64}	P_{72} P_{71} P_{73} P_{74}
Importance grade (I)	I_{11} I_{12} I_{13} I_{14}	I_{21} I_{22} I_{23} I_{24}	I_{31} I_{32} I_{33} I_{34}	I_{41} I_{42} I_{43} I_{44}	I_{51} I_{52} I_{53} I_{54}	I_{61} I_{62} I_{63} I_{64}	I_{71} I_{72} I_{73} I_{74}

above given the data assessed by operations managers. Therefore, the following section of this paper proposes an algorithm for evaluating the degree of MF for use by a group of decision-makers.

4. Evaluating the degree of manufacturing flexibility

A stepwise presentation of fuzzy multi-attribute two-stage MF evaluation algorithm as follows:

Step 1: Identify the environmental changes influencing manufacturing system, and then decide the flexibility dimensions (Xi) and related metrics (Xij).

Step 2: Determine the importance grade (Ii and Iij) for each Xi and Xij, respectively, and identify the performance rating (Pij) for each Xij.

Step 3: Aggregate Pij and Iij with respect to each Xij, and then convert them to obtain the fuzzy assessment matrix (M(Xi)) for each Xi, and convert Ii to obtain the fuzzy assessment vector (I1(Xi)) for each Xi.

Step 4: Aggregate M(Xi) to obtain the fuzzy assessment vector (F1(Xi)) for each Xi.

Step 5: Defuzzify I1(Xi) and F1(Xi) to obtain the aggregated weighted rating (D(Xi)) and importance (I(Xi)), respectively, and then calculate the difference (γ(Xi)) for each Xi.

Step 6: Aggregate I1(Xi) and F1(Xi) to obtain the fuzzy assessment vectors (I2(MF) and F2(MF)), respectively.

Step 7: Defuzzify I2(MF) and F2(MF) to obtain the degree of MF (D(MF)) and importance of MF (I(MF)), respectively, and then calculate the difference (γ(MF)) for MF, and determine the need and best directions for improving MF.

4.1 The first-stage assessment

Let P_{ij} and I_{ij} are linguistic labels, as in Table 1, which are established by operations managers. In order to deal with the management of various linguistic label sets, the evaluation values for each flexibility metrics should be transformed into a linguistic label set V, that a fuzzy assessment matrix for $X_i \times V$ can be formed. For instance, let $X_i = X_2$, and $w_1 = 0.5$. By the convex combination of two linguistic labels, the flexibility metrics of X_2 are X_{21}, X_{22}, X_{23}, X_{24} and the corresponding weighted rating of flexibility are $C^2(I_{21}, P_{21})$, $C^2(I_{22}, P_{22})$, $C^2(I_{23}, P_{23})$ and $C^2(I_{24}, P_{24})$, respectively, obtained from Table 2. Thus a fuzzy assessment matrix $M(X_2)$ obtained from Figure 1, as follows:

By the same way, we can form fuzzy assessment matrices $M(X_1)$, $M(X_3)$, $M(X_4)$, $M(X_5)$, $M(X_6)$ and $M(X_7)$ for X_1, X_3, X_4, X_5, X_6, X_7, respectively. Similarly, importance grade of flexibility dimensions are also transformed into fuzzy assessment vectors for $I_i \times V$, as follows:

$$M(X_2)= \begin{array}{c} \\ X_{21} \\ X_{22} \\ X_{23} \\ X_{24} \end{array} \begin{array}{c} \quad v_0 \qquad\qquad v_1 \qquad \cdots \qquad v_{10} \\ \begin{pmatrix} u(C^2(I_{21},P_{21}),v_0)u(C^2(I_{21},P_{21}),v_1)\ldots u(C^2(I_{21},P_{21}), v_{10}) \\ u(C^2(I_{22},P_{22}), v_0)u(C^2(I_{22},P_{22}),v_1)\ldots u(C^2(I_{22},P_{22}), v_{10}) \\ u(C^2(I_{23},P_{23}),v_0)u(C^2(I_{23},P_{23}),v_1)\ldots u(C^2(I_{23},P_{23}),v_{10}) \\ u(C^2(I_{24},P_{24}),v_0)u(C^2(I_{24},P_{24}),v_1)\ldots u(C^2(I_{24},P_{24}),v_{10}) \end{pmatrix} \end{array}.$$

$I1(Xi) = (u(Ii, v0), u(Ii, v1), \ldots, u(Ii, v10))$ *for* $i = 1, 2, \ldots, 7.$

Evaluating the fuzzy assessment vector for weighted rating of each X_i as follows: Let

$F1(Xi,vk) = \Phi\ Q1(u(C2(Ii1,Pi1),vk),u(C2(Ii2, Pi2),vk),u(C2(Ii3, Pi3),vk),u(C2(Ii4, Pi4),vk))$ *for* $i = 1, 2, \ldots, 7, k = 0, 1, \ldots, 10,$

where Φ_{Q1} is the MEOWA operator with the maximum entropy weighting vector W_1^*, obtained from the non-decreasing proportional linguistic quantifier Q_1, which represents the fuzzy majority over the flexibility metrics. Then, the fuzzy assessment vector for each X_i, $F1(X_i)$ is defined as

$F1(X_i) = (F1(X_i,v_0), F1(X_i,v_1),\ldots, F1(X_i,v_{10}))$ *for* $i = 1, 2, \ldots, 7.$

The aggregated weighting rating and importance for each X_i, $D(X_i)$ and $I(X_i)$ are defuzzified by the centroid method, respectively, as follows:

$$D(Xi) = \sum_{k=0}^{10} VG(k)\times F1(Xi,vk)/ \sum_{k=0}^{10} F1(Xi,vk) \text{ for } i = 1, 2, \ldots, 7,$$

$$I(Xi) = \sum_{k=0}^{10} VG(k)\times u(Ii,vk)/\sum_{k=0}^{10} u(Ii,vk) \text{ for } i = 1, 2, \ldots, 7.$$

Then computing the differences between the $I(X_i)$ and $D(X_i)$ with respect to each X_i, $\gamma(X_i)$ is defined as

$\gamma(Xi) = I(Xi) - D(Xi)$ *for* $i = 1, 2, \ldots, 7.$

Comparing $\gamma(X_i)$ for each X_i may yield a maximum positive value, which is represented the best directions for improving MF.

4.2 Second-stage assessment

The fuzzy assessment vector for weighted rating and importance of each X_i should be evaluated to determine the degree of MF. Using the concept of fuzzy majority over the flexibility dimensions specified by a linguistic quantifier Q_2, and applying the MEOWA operator Φ_{Q2} associated with the maximum entropy weighting vector W_2^*, to yields the fuzzy assessment vector for MF as follows: Let

$$I2(MF, v_k) = \Phi_{Q2}\,(u(I_1, v_k), u(I_2, v_k), \dots, u(I_7, v_k)) \text{ for } k = 0, 1, \dots, 10,$$

$$F2(MF, v_k) = \Phi_{Q2}\,(F1(X_1, v_k), F1(X_2, v_k), \dots, F1(X_7, v_k)) \text{ for } k = 0, 1, \dots, 10.$$

Then the fuzzy assessment vector for MF, F2(MF) and I2(MF) is defined as, respectively,

$$I2(MF) = (I2(MF, v_0), I2(MF, v_1), \dots, I2(MF, v_{10})),$$

$$F2(MF) = (F2(MF, v_0), F2(MF, v_1), \dots, F2(MF, v_{10})),$$

where the I2(MF) and F2(MF) represent the importance of MF and the degree of MF, respectively, according to the assessments of operation mangers.

The aggregated weighting rating and importance for MF, D(MF) and I(MF) are defuzzified by the centroid method, respectively, as follows:

$$D(MF) = \sum_{k=0}^{10} VG(k)\times F2(MF, v_k) / \sum_{k=0}^{10} F2(MF, v_k),$$

$$I(MF) = \sum_{k=0}^{10} VG(k)\times I2(MF, v_k) / \sum_{k=0}^{10} I2(MF, v_k).$$

Then computing the difference between the I(MF) and D(MF) with respect to MF, γ(MF) is defined as

$$\gamma(MF) = I(MF) - D(MF).$$

If γ(MF) is a positive value, then decision-makers should be improving MF.

5. Numerical example

Table 4. Assessing data

Flexibility dimension ('as many as possible')	X_1	X_2	X_3	X_4	X_5	X_6	X_7
Importance grade (GI)	H	VH	H	M	M	H	L
Flexibility metric (with 'Q_1')	X_{11} X_{12} X_{13} X_{14}	X_{21} X_{22} X_{23} X_{24}	X_{31} X_{32} X_{33} X_{34}	X_{41} X_{42} X_{43} X_{44}	X_{51} X_{52} X_{53} X_{54}	X_{61} X_{62} X_{63} X_{64}	X_{71} X_{72} X_{73} X_{74}
Performance rating (P)	M H H M	L L M L	VL L M L	H VH H H	DL L VL VL	VH DH H VH	M H L M
Importance grade (I)	VH H M VH	VH H M VH	VH H M VH	VH H M VH	VH H M VH	VH H M VH	VH H M VH

$C^2(I_{21}, P_{21}) = H$, $C^2(I_{22}, P_{22}) = M$,

$C^2(I_{23}, P_{23}) = M$, $C^2(I_{24}, P_{24}) = H$.

The following the two-stage assessment is applied to the case of a leading Taiwan firm in the bicycle industry is discussed. For reasons of confidentiality of the name of the firm is not revealed. measure the degree of MF. Managers have identified the importance grade and related performance rating, as presented in Table 4.

By the first-stage assessment: The corresponding $C^2(I,P)$ from Table 2, as follows:

From Figure 1 we obtain

$$M(X_2) = \begin{array}{c} \\ X_{21} \\ X_{22} \\ X_{23} \\ X_{24} \end{array} \begin{array}{c} \begin{array}{ccccccccccc} v_0 & v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 & v_9 & v_{10} \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0.5 & 1 & 1 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & 1 & 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & 1 & 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.5 & 1 & 1 & 0.5 & 0 & 0 \end{pmatrix} \end{array}.$$

Similarly, we have

$$M(X_1)=\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0.5 & 1 & 1 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.5 & 1 & 1 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.5 & 1 & 1 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.5 & 1 & 1 & 0.5 & 0 & 0 \end{pmatrix}, M(X_3)=\begin{pmatrix} 0 & 0 & 0 & 0 & 0.5 & 1 & 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & 1 & 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & 1 & 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.5 & 1 & 1 & 0.5 & 0 & 0 \end{pmatrix},$$

$$M(X_4)=\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 1 & 1 & 0.5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 1 & 1 & 0.5 \\ 0 & 0 & 0 & 0 & 0 & 0.5 & 1 & 1 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 1 & 1 & 0.5 \end{pmatrix}, M(X_5)=\begin{pmatrix} 0 & 0 & 0 & 0 & 0.5 & 1 & 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & 1 & 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 1 & 1 & 0.5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & 1 & 0.5 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$M(X_6)=\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 1 & 1 & 0.5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 1 & 1 & 0.5 \\ 0 & 0 & 0 & 0 & 0 & 0.5 & 1 & 1 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 1 & 1 & 0.5 \end{pmatrix}, M(X_7)=\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 1 & 1 & 0.5 \\ 0 & 0 & 0 & 0 & 0 & 0.5 & 1 & 1 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & 1 & 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 1 & 1 & 0.5 \end{pmatrix}, \text{and}$$

$$I1(I)=\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0.5 & 1 & 1 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 1 & 1 & 0.5 \\ 0 & 0 & 0 & 0 & 0 & 0.5 & 1 & 1 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & 1 & 0.5 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

$$
\begin{array}{ccccccccccc}
0 & 0 & 0 & 0 & 0.5 & 1 & 0.5 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.5 & 1 & 1 & 0.5 & 0 & 0 \\
0 & 0 & 0.5 & 1 & 1 & 0.5 & 0 & 0 & 0 & 0 & 0
\end{array}\Bigg)
$$

Using the linguistic quantifier 'as many as possible' with the pair (0.5, 1), the algorithm for calculating the maximum entropy weighting vector yields the weighting vector W_1, the orness α_1 and the maximum entropy weighting vector W_1^* as follows:

$W_1 = [0, 0, 0.5, 0.5]$, in which

$W_1(3) = Q(3/4) - Q(2/4) = ((0.75-0.5)/(1-0.5)) - ((0.5-0.5)/(1-0.5)) = 0.5$,

$\alpha_1 = ((4-1)\times 0 + (4-2)\times 0 + (4-3)\times 0.5 + (4-4)\times 0.5) / (4-1) = 0.1667$,

$W_1^* = [0.0311, 0.0856, 0.2355, 0.6478]$.

Then the fuzzy assessments for each flexibility dimension obtained are:

$F1(X_1) = (0, 0, 0, 0, 0, 0.5, 1, 1, 0.5, 0, 0)$,

$F1(X_2) = (0, 0, 0, 0, 0.0584, 0.5584, 0.5584, 0.1167, 0.0584, 0, 0)$,

$F1(X_3) = (0, 0, 0, 0, 0.1761, 0.6761, 0.5156, 0.0311, 0.0156, 0, 0)$,

$F1(X_4) = (0, 0, 0, 0, 0, 0.0156, 0.0311, 0.5156, 0.6761, 0.3522, 0.1761)$,

$F1(X_5) = (0, 0, 0.0156, 0.0311, 0.5156, 0.6761, 0.1761, 0, 0, 0, 0)$,

$F1(X_6) = (0, 0, 0, 0, 0, 0.0156, 0.0311, 0.5156, 0.6761, 0.3522, 0.1761)$,

$F1(X_7) = (0, 0, 0, 0, 0.0156, 0.5156, 0.6761, 0.3522, 0.1761, 0, 0)$,

where, for example, the value $F1(X_2, v_6)$ is obtained according to this expression:

$F1(X_2, v_6) = \Phi_{Q1}(1, 0.5, 0.5, 1) = 0.5584$.

Defuzzified by the centroid method, let

$D(X_2) = (0\times 0.0333+0\times 0.1+0\times 0.2+0\times 0.3+0.0584\times 0.4+0.5584\times 0.5+0.5584\times 0.6+0.1167\times 0.7+0.0584\times 0.8 +0\times 0.9+ 0\times 0.9667)/(0.0584+0.5584+0.5584+0.1167+0.0584) = 0.5673$.

Similarly, we have

$D(X_1)=0.65$, $D(X_3)=0.5317$, $D(X_4)=0.8012$, $D(X_5)=0.4683$, $D(X_6)=0.8012$, $D(X_7)=0.6091$.

By the same way, we have

$I(X_1)=0.65$, $I(X_2)=0.8444$, $I(X_3)=0.65$, $I(X_4)=0.5$, $I(X_5)=0.5$, $I(X_6)=0.65$, $I(X_7)=0.35$.

The differences of each dimension as shown in Table 5, and then determine dimension X2 has a maximum positive value.

By the second-stage assessment: Using the linguistic quantifier 'as many as possible' with the pair (0.5, 1), yields the weighting vector W2, the orness α 2 and the maximum entropy weighting vector W2*, as follows:

W_2= [0, 0, 0, 0.1429, 0.2857, 0.2857, 0.2857],

α_2=0.2143,

W_2^*= [0.0187, 0.0313, 0.0525, 0.0880, 0.1476, 0.2473, 0.4145].

Then the fuzzy assessment vectors for MF obtained are:

I2(MF) = (0,0,0.0094,0.0187,0.0606,0.3178, 0.2204, 0.1466,0.1047,0.0187,0.0094),

F2(MF) = (0,0,0.0003,0.0006,0.0196,0.1926, 0.1612, 0.1179, 0.0881, 0.0176, 0.0088),

where, for example, the value F2(MF, v_6) is obtained according to this expression:

F2(MF, v_6)=Φ_{Q2}(0.5, 0.5584, 0.6761, 0.0156, 0.6761, 0.0156, 0.5156) = 0.1926.

Table 5. The result of first-stage assessment

Flexibility dimension	X_1	X_2	X_3	X_4	X_5	X_6	X_7
(1) Importance grade	0.6500	0.8444	0.6500	0.5000	0.5000	0.6500	0.3500
(2) Aggregated Weighted rating	0.6500	0.5673	0.5317	0.8012	0.4683	0.8012	0.6091
Difference (1) − (2)	0	+0.2771	+0.1183	-0.3012	+0.0317	-0.1512	-0.2519

Table 6. The combination of the Performance rating (P) and the importance grade (I)

Importance grade (I)		Performance rating (P): Low → High DL , VL	L , M	H	VH , DH
Low	DL VL	(b)	(c)	(d)	(e)
↓	L M	(f)	(g)	(h)	(i)
	H	(j)	(k)	(l)	(m)
High	VH DH	(n)	(o)	(p)	(q)

Defuzzified by the centroid method, let

D(MF) = (0×0.0333+0×0.1+0.0003×0.2+0.0006×0.3+0.0196×0.4+0.1926×0.5+ 0.1612×0.6 +0.1179×0.7+0.0881×0.8+0.0176×0.9+0.0088×0.9667)/(0.0003+

0.0006+0.0196+0.1926+0.1612+0.1179+0.0881+0.0176+0.0088) = 0.6238.

By the same way, we have I(MF) = 0.5905. Therefore, 0.5905 and 0.6238 are the importance and the degree of MF, respectively, for this case.

Then the difference for MF is obtained, as follows,

γ(MF) = －0.0333.

Therefore, decision-maker should not be improving MF.

Table 7 The combination of importance grade (I), performance rating (P) and degree of manufacturing flexibility (D(MF))

Dimension	X_1				X_2				X_3				X_4				X_5				X_6				X_7				D(MF)
Metric	X_{11}	X_{12}	X_{13}	X_{14}	X_{21}	X_{22}	X_{23}	X_{24}	X_{31}	X_{32}	X_{33}	X_{34}	X_{41}	X_{42}	X_{43}	X_{44}	X_{51}	X_{52}	X_{53}	X_{54}	X_{61}	X_{62}	X_{63}	X_{64}	X_{71}	X_{72}	X_{73}	X_{74}	
(a) I(1)	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0.0555
P(1)	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
(b) I(1, 2)	1	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1	1	1	0.1552
P(1, 2)	1	2	1	2	1	2	1	2	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	1	1	2	2	
(c) I(1, 2)	1	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1	1	1	0.2911
P(3, 4)	3	4	3	4	3	4	3	4	3	3	3	3	3	3	3	3	4	4	4	4	4	4	4	4	3	3	4	4	
(d) I(1, 2)	1	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1	1	1	0.4184
P(5)	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	
(e) I(1, 2)	1	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1	1	1	0.5288
P(6, 7)	7	6	7	6	7	6	7	6	7	7	7	7	7	7	7	7	6	6	6	6	6	6	6	6	7	7	6	6	
(f) I(3, 4)	4	4	4	4	3	3	3	3	4	4	4	4	3	3	3	3	4	4	4	4	3	3	3	3	4	4	4	4	0.3083
P(1, 2)	1	2	1	2	1	2	1	2	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	1	1	2	2	
(g) I(3, 4)	4	4	4	4	3	3	3	3	4	4	4	4	3	3	3	3	4	4	4	4	3	3	3	3	4	4	4	4	0.4712
P(3, 4)	4	3	4	3	4	3	4	3	3	3	3	3	3	3	3	3	4	4	4	4	4	4	4	4	3	3	4	4	
(h) I(3,4)	4	4	4	4	3	3	3	3	4	4	4	4	3	3	3	3	4	4	4	4	3	3	3	3	4	4	4	4	0.5816
P(5)	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	
(i) I(3,4)	4	4	4	4	3	3	3	3	4	4	4	4	3	3	3	3	4	4	4	4	3	3	3	3	4	4	4	4	0.7089
P(6, 7)	7	6	7	6	7	6	7	6	7	7	7	7	7	7	7	7	6	6	6	6	6	6	6	6	7	7	6	6	
(j) I(5)	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	0.4256

P(1, 2)	1	2	1	2	1	2	1	2	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	1	1	2	2	
(k) I(5)	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	0.5634
P(3, 4)	3	4	3	4	3	4	3	4	3	3	3	3	3	3	3	3	4	4	4	4	4	4	4	4	4	4	3	3	
(l) I(5)	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	0.6500
P(5)	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	
(m) I(5)	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	0.8444
P(6, 7)	7	6	7	6	7	6	7	6	7	7	7	7	7	7	7	7	6	6	6	6	6	6	6	6	7	7	6	6	
(n) I(6, 7)	7	7	7	7	6	6	6	6	7	7	7	7	6	6	6	6	7	7	7	7	6	6	6	6	7	7	7	7	0.5288
P(1, 2)	1	2	1	2	1	2	1	2	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	1	1	2	2	
(o) I(6, 7)	7	7	7	7	6	6	6	6	7	7	7	7	6	6	6	6	7	7	7	7	6	6	6	6	7	7	7	7	0.7089
P(3, 4)	3	4	3	4	3	4	3	4	3	3	3	3	3	3	3	3	4	4	4	4	4	4	4	4	3	3	4	4	
(p) I(6, 7)	7	7	7	7	6	6	6	6	7	7	7	7	6	6	6	6	7	7	7	7	6	6	6	6	7	7	7	7	0.8444
P(5)	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	
(q) I(6, 7)	7	7	7	7	6	6	6	6	7	7	7	7	6	6	6	6	7	7	7	7	6	6	6	6	7	7	7	7	0.9445
P(6, 7)	7	6	7	6	7	6	7	6	7	7	7	7	7	7	7	7	6	6	6	6	6	6	6	6	7	7	6	6	
(r) I(7)	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	0.9445
P(7)	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	

6. Sensitivity analysis of the two-stage MF assessment

According to the monotonic property, in this paper some examples are thus presented to shown that higher the combination of importance grade and performance rating the higher the degree of MF.

Table 8 Degree of manufacturing flexibility in three ways table (importance grade, performance rating and linguistic quantifier)

		Performance rating (P)			
Importance grade (I)	Linguistic quantifier	(DL, VL) Low	(L, M) Middle low	(H) Middle high	(VH, DH) High
(DL, VL) Low	As many as possible	0.1552	0.2911	0.4184	0.5288

(L, M) Middle low	As many as possible	0.3083	0.4712	0.5816	0.7089
(H) Middle high	As many as possible	0.4256	0.5634	0.6500	0.8444
(VH, DH) High	As many as possible	0.5288	0.7089	0.8444	0.9445

Since the importance grade and related performance rating are corresponding in ranks, we divide them into four classes as the corresponding Table 8, then Table 9 can be obtained.

In order to show the monotonic property, suppose that decision-makers assign a fixed linguistic quantifier to the corresponding flexibility dimensions and metrics. Then the combination of the performance rating, the importance, and related degree of MF are presented in Table 10, which from (a) to (r). Moreover, Table 11 represented the degree of MF of each entry of Table 9, and show that all the entries on the diagonals from the upper left to the lower right are increasing. Also the entries from the top to the bottom and from the left to right are non-decreasing. It conceded with what we expected.

7. Conclusion

The structure model used to evaluate the degree of manufacturing flexibility, is very useful in manufacturing system development. The importance grades or performance ratings must be improved until acceptable when evaluating the degree of manufacturing flexibility. If the degree of manufacturing flexibility is too low, it may have to be improved. The dimensions of manufacturing flexibility on which improvements must best be made should be determined. Issues of practical importance follow.

(1) In general, if an operation manager wants to estimate the degree of flexibility in a manufacturing system, he/she must be invited to participate in a group of evaluators whose collective experience extends across a broad range of manufacturing organizations. Their opinions should be reasonable and unambiguous.

(2) Measuring manufacturing flexibility is strategically important, and must affect the formation of manufacturing strategy, to ensure that a manufacturing system can cope with environmental changes.

(3) This model can be run on a computer routinely or at any time as required.

References

1. Beach, R., A. P. Muhlemann, D. H. R. Price, A. Paterson, and J. A. Sharp, "A review of manufacturing flexibility," European Journal of Operational Research, Vol.122, pp.41-57 (2000).

2. Delgado, M., J. L. Verdegay, and M. A. Vila, “On aggregation operations of linguistic labels,” International Journal of Intelligent Systems, Vol.8, pp.351-370 (1993).

3. De Toni, A., and S. Tonchia, “Manufacturing flexibility: a literature review,” International Journal of Production Research, Vol.36, No.6, pp.1587-1617 (1998).

4. Filev, D., and R. R. Yager, “Analytic properties of maximum entropy OWA operators.,” Information Sciences, Vol.85, pp.11-27 (1995).

5. Gerwin, D., “Manufacturing flexibility: a strategic perspective,” Management Science, Vol.39, No.4, pp.395-410 (1993).

6. Golden, W., and P. Powell, “Towards a definition of flexibility: in search of the Holy Grail,” Omega, Vol.28, pp.373-384 (2000).

7. Gupta, Y. P., and S. Goyal, “Flexibility of the manufacturing system: concepts and measurement,” European Journal of Operational Research, Vol.43, pp.119-135 (1989).

8. Herrera, F., Herrera-Viedma, E., “Linguistic decision analysis: steps for solving decision problems under linguistic information,” Fuzzy Sets and Systems, Vol.115, pp.67-82 (2000).

9. Herrera, F., E. Herrera-Viedma, and L. Martinez, “A fusion approach for managing multi-granularity linguistic term sets in decision making,” Fuzzy Sets and Systems, Vol.114, pp.43-58 (2000).

10. Herrera, F., Herrera-Viedma, E., Verdegay, J. L., “A sequential selection process in group decision making with a linguistic assessment approach,” Information Sciences, Vol.85, pp.223-239 (1995).

11. Mitchell, H. B., Estrakh, D. D., “A modified OWA operator and its use in lossless DPCM image compression,” International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems, Vol.5(4), pp.429-436 (1997).

12. O’Hagan, M., “Aggregating template rule antecedents in real-time expert systems with fuzzy set logic,” Proceedings 22nd annual IEEE Asilomar Conference on Signals, Systems and Computers, Pacific Grove, CA (1988).

13. Sarker, B. R., S. Krishnamurthy, and S. G. Kuthethur, “A survey and critical review of flexibility measures in manufacturing systems,” Production Planning and Control, Vol.5, No.6, pp.512-523 (1994).

14. Sethi, A. K., and S. P. Sethi, “Flexibility in manufacturing: a surrey,” International Journal of Flexible Manufacturing Systems, Vol.2, No.4, pp.289-328 (1990).

15. Slack, N., “The flexibility of manufacturing systems,” International Journal of Operations

and Production Management, Vol.7, No.4, pp.35-45 (1987).

16. Upton, D. M., “The management of manufacturing flexibility,” California Management Review, Vol.36, No.2, pp.72-89 (1994).
17. Vokurka, R. J., and S. W. O’Leary-Kelly, “A review of empirical research on manufacturing flexibility,” Journal of Operations Management, Vol.18, pp.485-501 (2000).
18. Wang, R. C., and S. J. Chuu, “Group decision-making using a fuzzy linguistic approach for evaluating the flexibility in a manufacturing system,” European Journal of Operational Research, Vol.154, pp.363-572 (2004).
19. Yager, R. R., “On ordered weighted averaging aggregation operators in multi-criteria decision making,” IEEE Transactions on Systems, Man and Cybernetics, Vol.18, pp.183-190 (1988).
20. Zadeh, L. A., “Fuzzy set,” Information and Control, Vol.8, pp.338-353 (1965).
21. Zadeh, L. A., “The concept of a linguistic variable and its application to approximate reasoning,” Information Sciences, Vol.8, pp.199-249 (I); pp.301-357 (II); Vol.9, pp.43-80 (III) (1975).